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SINGLE SERVER RETRIAL QUEUEING MODELS OF MULTIPLE VACATION'S WITH ENCOURAGED CUSTOMERS

Vinita Yadav*
E mail - viniyadav6598@gmail.com
Research Scholar, Department of Mathematics, Baba Mastnath University, Asthal Bohar, Rohtak-124021(Haryana)

Naveen Kumar
Professor, Department of Mathematics, Baba Mastnath University, Asthal Bohar, Rohtak-124021(Haryana)
Email: naveenkapilrtk@gmail.com

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Abstract
The single server queueing model of finite size with repeated vacations and promoted customers arrival is examined in this work. By employing the recursive approach, the steady-state solution is achieved. When the server is idle, or the system is empty, the server takes a vacation. He will resume regular work once his vacation is over if he discovers any customers waiting for service; if not, he will take another vacation and so on. By using a recursive method, some of the system's operational characteristics, such as the predicted queue length, sojourn time, and probability of various server statuses, are determined. When businesses provide attractive off-season sales or holiday season discounts, the number of consumers suddenly jumps, giving birth to the term encouraged customers.
Keywords: Vacation, Encouraged customers, Performance metric, Poisson distribution and Queuing model.

1. Introduction

Applications of queuing theory are numerous and include telecommunications, city traffic, the medical area, inventory and management, and more. A.K. Erlang's groundbreaking contributions to the discipline have earned him the title of father of queueing theory. Levy & Yechiali talked about a queuing model with holidays [1]. Doshi [2] also investigated holidays in a waiting system. Later, vacation models were examined by Chatterjee [3] and Takagi [4]. Baba [6] examined the G1/M/1 queue that has several vacations. Afterwards, In recent years, academics have become interested in queuing systems with vacations. A queueing model incorporating working vacations was developed by Servi and Finn [7]. Vacation queueing models were examined by N. Tian et al. [8]. J.C. Ke [9] later researched further advancements in vacation queueing models.

In general, there are two kinds of vacation queueing models: single and multiple vacation queueing models. Regardless of how many customers are in the system, the server in a single vacation queueing model returns to the regular service state at the conclusion of the vacation, but in multiple vacation queueing models, if the server detects no customers in the queue, he keeps on taking successive vacations till a consumer is in queue. Readers can see [10] and the references therein for these. This research examines a finite capacity, multiple vacation, single server queueing model with encouraged consumers. To the best of the author's knowledge, a few research has been done on the queueing model with finite capacity with repeated vacations and encouraged customers attendance.

Five sections make up the remainder of the paper. The model's mathematical explanation is covered in Section 2. The model's differential-difference equations are shown in Section 3. Section 4 discusses the model's steady-state equations. A few of the system's performance metrics are explained in Section 5. Section 6 presents the numerical results. Section 7 presents the conclusion and future scope.

2 Description of the Model

The assumptions that follow are used to investigate a finite-size, single server Markovian queueing model with multiple vacations.

a. Customers come accordance to the Poisson distribution with parameter $\mu = \lambda(1+\xi)$, where ξ is the factor that determines the customers' encouraged arrival. It is assumed that the service rate is exponentially distributed.

b. Customers receive attention to on a First Cum First Serve (FCFS) basis.

c. If the system is empty, the single server enjoys a vacation. With parameter θ, the vacation time divides exponentially. The server only returns normal activities when his break is over if there remain customers waiting in the system; otherwise, he will take another holiday.

d. The system's finite size is M.

The system's transition state diagram is shown in Figure 1.

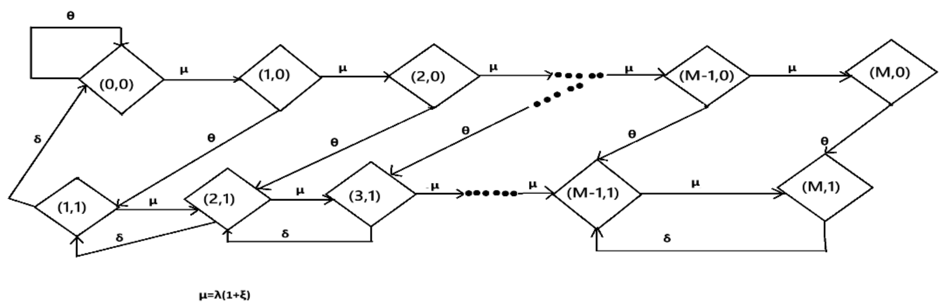

Fig.1

Assume S(t) is the server place at time t and N(t) is the total number of consumers in the system at time t.

$$S(t) = \begin{cases} 0, \text{if the server is on vacation} \\ 1, \text{if the server is in a normal working state} \end{cases}$$

Then, $\{N(t), S(t), t \geq 0\}$ is a Markov process, and $\Omega = \{(m, j), 1 < m \leq M, j = 0, 1\} \cup \{(0, 0)\}$is an example of its state space.

3. Differential Difference Equations

The model's differential difference equations are:

$$\frac{d}{dt}p_{0,0}(t) = \delta p_{1,1}(t) - \lambda(1+\xi)p_{0,0}(t) \quad (1.1)$$

$$\frac{d}{dt}p_{m,0}(t) = \lambda(1+\xi)p_{m-1,0}(t) - (\lambda(1+\xi) + \theta)p_{m,0}(t), \; m = 1,2,3, \dots \dots .M-1 \quad (1.2)$$

$$\frac{d}{dt}p_{M,0}(t) = \lambda(1+\xi)p_{M-1,0}(t) + \theta p_{M,0}(t) \quad (1.3)$$

$$\frac{d}{dt}p_{1,1}(t) = \delta p_{2,1}(t) + \theta p_{1,0}(t) - (\lambda(1+\xi) + \delta)p_{1,1}(t) \quad (1.4)$$

$$\frac{d}{dt}p_{m,1}(t) = \delta p_{m+1,1}(t) + \lambda(1+\xi)p_{m-1,1}(t) + \theta p_{m,0}(t) - (\lambda(1+\xi) + \delta)p_{m,1}(t),$$
$$m = 2,3, \dots \dots .M-1 \quad (1.5)$$

$$\frac{d}{dt}p_{M,1}(t) = \delta p_{M-1,1}(t) + \theta p_{M,0}(t) - \delta p_{M,1}(t) \quad (1.6)$$

4. Steady- State Equations and Solutions

Taking limit $t \to \infty$, we have

$\lim_{t\to\infty} p_{m,j}(t) = p_{m,j}$, j=0,1

$\frac{d}{dt}p_{m,j}(t) = 0$, j=0,1

$$\lambda(1+\xi)p_{0,0} = \delta p_{1,1} \quad (1.7)$$

$$(\lambda(1+\xi)+\theta)p_{m,0} = \lambda(1+\xi)p_{m-1,0},\ m = 1,2,3,\dots\dots M-1 \quad (1.8)$$

$$\theta p_{M,0} == \lambda(1+\xi)p_{M-1,0} \quad (1.9)$$

$$(\lambda(1+\xi)+\delta)p_{1,1} = \delta p_{2,1} + \theta p_{1,0} \quad (1.10)$$

$$(\lambda(1+\xi)+\delta)p_{m,1} = \delta p_{m+1,1} + \lambda(1+\xi)p_{m-1,1} + \theta p_{m,0},\ m = 1,2,3,\dots\dots M-1 \quad (1.11)$$

$$\delta p_{M,1} = \delta p_{M-1,1} + \theta p_{M,0} \quad (1.12)$$

On solving equation (1.8) recursively, we get:

$$p_{m,0} = \left[\frac{(\lambda(1+\xi))}{\lambda(1+\xi)+\theta}\right]^m p_{0,0} \qquad m = 1,2,3,\dots\dots M-1 \quad (1.13)$$

By equation 1.9, we have

$$p_{m,0} = \frac{\lambda(1+\xi)}{\theta}\left[\frac{(\lambda(1+\xi))}{\lambda(1+\xi)+\theta}\right]^{M-1} p_{0,0} \quad (1.14)$$

By equations (1,7) (1,10) (1,11), we get

$$p_{1,1} = \left(\frac{\lambda(1+\xi)}{\delta}\right) p_{0,0} \quad (1.15)$$

$$p_{m,1} = \left[\left(\frac{\lambda(1+\xi)}{\delta}\right)^m \left(1+\frac{(\lambda(1+\xi))^2\delta+\lambda(1+\xi)\theta\delta-\lambda(1+\xi)\delta^2-\delta^2\theta}{(\lambda(1+\xi)-\delta)(\lambda(1+\xi)+\theta)(\lambda(1+\xi)+\theta-\delta)}\right) - \left(\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right)^m \frac{(\lambda(1+\xi)+\theta)}{(\lambda(1+\xi)+\theta-\delta)}\right] p_{0,0}$$

$$m = 1,2,3,\dots\dots M-1 \quad \mathbf{(1.16)}$$

$$p_{M,1} = \left[\left(\frac{\lambda(1+\xi)}{\delta}\right)\left\{\left(\frac{\lambda(1+\xi)}{\delta}\right)^{M-1}\left(1+\frac{(\lambda(1+\xi))^2\delta+\lambda(1+\xi)\theta\delta-\lambda(1+\xi)\delta^2-\delta^2\theta}{(\lambda(1+\xi)-\delta)(\lambda(1+\xi)+\theta)(\lambda(1+\xi)+\theta-\delta)}\right) - \left(\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right)^{M-1}\frac{(\lambda(1+\xi)+\theta)}{(\lambda(1+\xi)+\theta-\delta)}\right\} + \frac{\lambda(1+\xi)}{\delta}\left(\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right)^{M-1}\right] p_{0,0} \quad (1.17)$$

Consequently, $p_{0,0}$, which can be constructed from the standardization condition as follows, is used to signify all of the probabilities.

$$\sum_{m=0}^{M} p_{m,0} + \sum_{m=1}^{M} p_{m,1} = 1 \quad (1.18)$$

$$p_{0,0}\left[1+\sum_{m=1}^{M-1}\left[\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right]^m + \frac{\lambda(1+\xi)}{\theta}\left[\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right]^{M-1} + \frac{\lambda(1+\xi)}{\delta} + \sum_{m=2}^{M}\left[\frac{\lambda(1+\xi)}{\delta}\right]^m\left(1+\frac{(\lambda(1+\xi))^2\delta+\lambda(1+\xi)\theta\delta-\lambda(1+\xi)\delta^2-\delta^2\theta}{(\lambda(1+\xi)-\delta)(\lambda(1+\xi)+\theta)(\lambda(1+\xi)+\theta-\delta)}\right) - \left(\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right)^m\frac{(\lambda(1+\xi)+\theta)}{(\lambda(1+\xi)+\theta-\delta)}\right] = 1 \quad (1.19)$$

$$p_{0,0} = \left[1+\sum_{m=1}^{M-1}\left[\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right]^m + \frac{\lambda(1+\xi)}{\theta}\left[\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right]^{M-1} + \frac{\lambda(1+\xi)}{\delta} + \sum_{m=2}^{M}\left[\frac{\lambda(1+\xi)}{\delta}\right]^m\left(1+\frac{(\lambda(1+\xi))^2\delta+\lambda(1+\xi)\theta\delta-\lambda(1+\xi)\delta^2-\delta^2\theta}{(\lambda(1+\xi)-\delta)(\lambda(1+\xi)+\theta)(\lambda(1+\xi)+\theta-\delta)}\right) - \left(\frac{\lambda(1+\xi)}{\lambda(1+\xi)+\theta}\right)^m\frac{(\lambda(1+\xi)+\theta)}{(\lambda(1+\xi)+\theta-\delta)}\right]^{-1} \quad (1.20)$$

5. Performance Measures

$E[L_{q_0}]$ =Expected queue length when server is on vacation.

$$=\sum_{m=0}^{M} mp_{m,0}$$

$$=p_{1,0} + 2p_{2,0} + 3\,p_{3,0} + \cdots\dots\dots + Mp_{M,0} \quad (1.21)$$

$E[L_{q_1}]$ = Expected queue length when server is on working state.

$=\sum_{m=1}^{M}(m-1)p_{m,1}$

$=p_{2,1}+2p_{3,1}+3p_{4,1}+\ldots\ldots+(M-1)\,p_{M,1}$ (1.22)

P_W = Probability that server is on working state

$=\sum_{m=1}^{M} p_{m,1}$ (1.23)

P_v = Probability that server is on vacation.

$=\sum_{m=0}^{M} p_{m,0}$ (1.24)

$E[L_q]$ = Expected queue length

$= E[L_{q_0}] + E[L_{q_1}]$ (1.25)

$E[W_q]$ = Expected sojourn time of customer in the queue.

$= \frac{EL_q}{\lambda(1+\xi)}$, (Using little's formula) (1.26)

EL_s = Expected system length

$=\sum_{m=0}^{M} mp_{m,0}+\sum_{m=1}^{M} mp_{m,1}$ (1.27)

EW_s = Expected sojourn time of customer in the system

$= \frac{EL_s}{\lambda(1+\xi)}$, (Using little's formula) (1.28)

6. Numerical Results

This section examines how the system's key performance metrics vary depending on a number of different factors. Unless they are considered variables to examine the changes, we have set the values as λ = 3, δ = 6, ξ = 0.5, θ = 0.30, and M = 62.

Sensitivity Analysis

Table 1: Effect of service rate δ on expected queue length EL_q for various values of ξ

S. No	Δ		E[L_q]	
		ξ =0.3	**ξ =0.4**	**ξ =0.5**
1	6.0	11.7084	12.5296	13.8524
2	6.3	11.6698	12.4804	13.8216
3	6.6	11.6393	12.4423	13.7967
4	6.9	11.6148	12.4125	13.7764
5	7.2	11.5952	12.3891	13.7598
6	7.5	11.5794	12.3707	13.746
7	7.8	11.5666	12.3562	13.7346
8	8.1	11.5563	12.344	13.7252
9	8.4	11.5479	12.3360	13.7173
10	8.7	11.5412	12.3291	13.7107

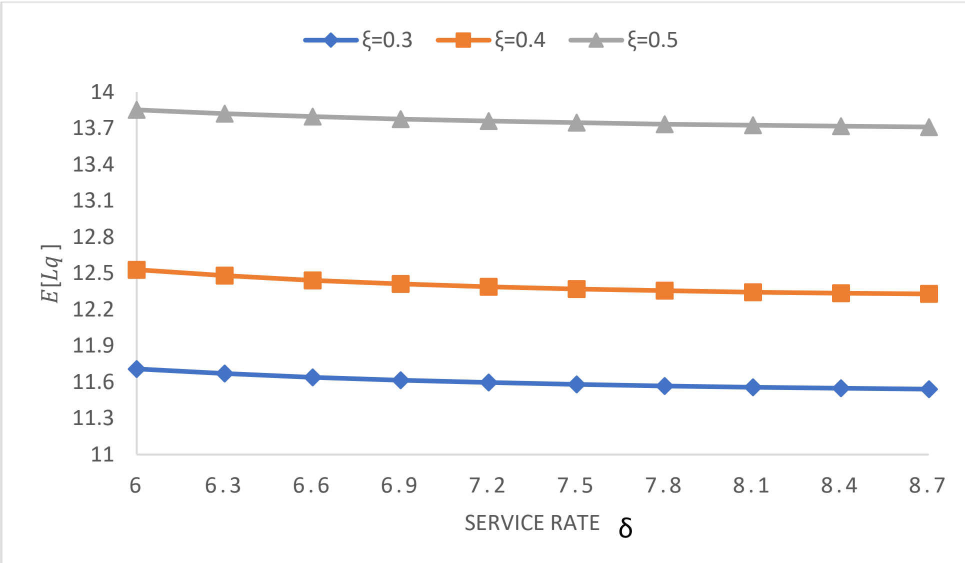

Fig.2 Expected queue length vs. δ for different ξ

Table 2: Effect of λ on expected queue length EL_q

S. No	Λ	$E[L_q]$ ξ =0.3	ξ =0.4	ξ =0.5
1	1.0	9.0708	9.7296	10.8245
2	1.2	9.698	10.4801	11.3216
3	1.4	10.2293	11.0058	11.9767
4	1.6	10.7748	11.4125	12.4774
5	1.8	11.3252	12.1591	12.9998
6	2.0	11.8594	12.7073	13.5746
7	2.2	12.3766	13.2569	14.1346
8	2.4	12.8563	13.9774	14.7225
9	2.6	13.3479	14.2860	15.1773
10	2.8	13.8412	14.3291	15.7107
11	3.0	14.3213	15.2314	16.3245
12	3.2	14.7850	15.7855	16.8731
13	3.4	15.2433	16.2890	17.5328

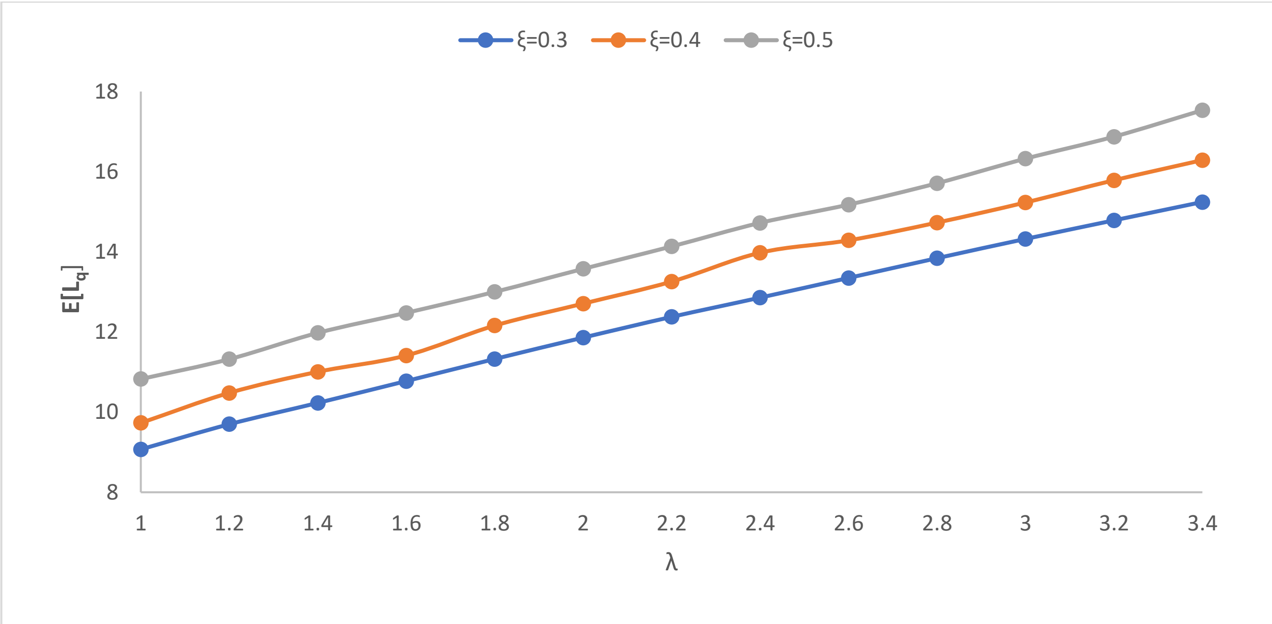

Fig.3 Expected queue length $E[L_q]$ vs. λ for different ξ

Because of the related drop in the server's mean service time, Figure 2 illustrates how, with a set encouragement rate ξ, the expected length of the queue continues to fall as μ increases. As the rate of encouragement increases, the expected length of the queue continues to grow.

Figure 3 shows how the average queue length changes as λ changes. When λ increases, the mean arrival rate also rises, provided that the encouragement rate remains constant. With the pace of encouragement, this expected line length increases to grow.

7. CONCLUSION AND FUTURE SCOPE

In a steady state, a finite capacity single server queueing model with repeated vacations and promoted customer arrival is examined. The recursive method has been used to obtain the formulas for expected system length and queue length in both regular and vacation states, as well as the estimated sojourn time of customers in the queue.

It has been found that the expected length of wait increases with the arrival rate and falls as the service and vacation rate increases. The effect of the vacation and arrival rates on the system's various state probabilities is examined both graphically and numerically. Any organization may benefit greatly from the results of this article, which show that vacations increase server efficiency and that special offers encourage consumers.

Additionally, systems with several servers and those in a temporary state can also benefit from studying this approach.

REFERENCES

1. Levy, Y., & Yechiali, U. (1975). Utilization of idle time in an M/G/1 queueing system. *Management Science*, *22*(2), 202-211.
2. Doshi, B. T. (1986). Queueing systems with vacations—a survey. *Queueing systems*, *1*, 29-66.
3. Chatterjee U., Mukerjee S.P. (1990). G1/M/1 queue with server vacations, Journal of the Operational Research Society, 41(1),83-87.
4. Takagi, H. (1994). Queueing analysis of polling models: progress in 1990–1993. *Frontiers in Queueing: Models, Methods and Problems*.

5. Gray, W. J., Wang, P. P., & Scott, M. (2000). A vacation queueing model with service breakdowns. *Applied Mathematical Modelling*, *24*(5-6), 391-400.
6. Baba Y., (1990). Analysis of a G1/M/1 queue with multiple working vacations, Operations Research Letters, 33(2),201-209.
7. Servi L.D., Finn S.G. (2002).M/M/1 queues with working vacations (M/M/1/W/V), Perform Evaluation 50, (41-52).
8. Tian, N., & Zhang, Z. G. (2006). *Vacation queueing models: theory and applications* (Vol. 93). Springer Science & Business Media.
9. Ke, J. C., Wu, C. H., & Pearn, W. L. (2013). A heuristic algorithm for the optimization of a retrial system with Bernoulli vacation. *Optimization*, *62*(3), 299-321.
10. Gupta, R. and Malik, S. (2021). Study of feedback queuing system with unreliable waiting server under multiple differentiated vacation policy, Ratio Mathematica, Vol. 40, pp. 146-161, 2021.