© UNIVERSAL RESEARCH REPORTS | Refereed | Peer Reviewed | Indexed
ISSN : 2348 - 5612 | Volume : 04 , Issue : 06 | July - September 2017

Algebric Graph Theory-A Brief Study

Mrs. Rckha Rani Asstt. Prof. in Mathematics
Govt.P.G College for Women,Rohtak

ISSN : 2348-5612 © URR
9 770234 856124

Algebric graph theory is that branch of graph theory where algebraic techniques are used to study graphs. In this branch, properties about graph are being translated into algebraic properties and then by making use of algebraic methods, theorems on graphs are deduced. The widely applied part of algebra to graph theory is linear algebra comprising of the theory of matrices and linear vector spaces. A graph is completely determined either by its adjacencies or incidences. This information can be conveniently stated in the matrix form.

The concept of graph theory came into existence during the first half of the 18th century. Since graph can be used to represent almost any physical problem involving discrete arrangements of objects and a relationship among them, the theory became a vbery natural and powerful tool in Mathematical Research. In general, any mathematical object involving points and connections between them can be called a graph; The study of graph theory was in a dormant state until the second half of 19th century. This theory stared to develop in an organized way in the first half of the 20th century. A large part of graph theory arose from the consideration of various games and recreational problems. Because of its simplicity graph theory has a wide range of applications in numerous other areas of physics, chemistry, communication science, computer technology, electrical and civil engineering, architecture, operational research, genetics, psychology, sociology, economics, anthropology and linguistics.

Graphs can be used to model a wide variety of real life problems. These problems can be studied and possibly can be solved with the help of graphs. For instance, a network of cities, which are represented by vertices can be modeled by graph by connecting the cities. In computer science, graph are used to represent networks of communications, data organizations etc. For instance, the link structure of a website can be represented by a directed graph, in which the web pages represents the vertices and each edge represent the direction from one page to another.

The molecules in chemistry and physics can also be studied by graphs. In chemistry, the structure of a molecule can be represented by means of a graph by referring the vertices as atoms and edges as bonds. Thus, graphs can serve as mathematical models to solve an appropriate graph-theoretic problem, and then interpret the solution in terms of the original problem. Graph theory is used in sociology also. Likewise graph theory is useful in biology and conservation efforts where the vertices represent the regions where certain species exist and edges represent the migration paths. Modern abstract algebra also plays a very powerful tool in the theory of graph as well as in its applications. Several algebraic structures like matrices, vector spaces, groups, rings and posets have acquired an important place in the analysis of graph theory. Some of the most difficult problems of graph theory like graph isomorphism and graph enumeration also fall in this domain. We refer this area of graph theory as algebraic graph theory.

In the next section we give the historical background of graph theory and also give a survey of some early results on graph.

The theory of graphs was motivated by Euler. In 1736, Euler settled a famous unsolved problem of his day called Konigsberg Bridge Problem, which subsequently led to the concept of an Eulerian graph and also the solution to this problem formed a firm basis of graph theory. After Euler, the theory of graph was further development by Kirchoff in 1847 and by Cayley. The work of Cayley was related to the studies of chemical composition, of saturated hydrocarbons with a given number of carbon atoms forming a tree. The problem of Cayley can be restated in graph theoretic terms as the problem of finding the number of trees with *p* vertices, in which no vertex has degree exceeding four. The notion of having a relationship between chemistry and algebra

was firs drawn by Sylvester in 1878. * The invention of a game in 1859 by Sir Hamilton'an, involving « solid dodecahedron, resulted the concept of Hamiltonian graph, while the planar graph was the outcome ot the investigation of polyhedral. Kuratowski revived this subject and obtained a criterion for a graph to bo planar while Whiteny s approach to planarity was based on the notion of duality.

Another important aspect is graph coloring. One of the most famous and productive problems in coloring of graphs is the 'four color conjecture* which states that any map drawn in the plane or the surface of the sphere can be colored with four colors.' This problem was first posed by Francis Guthne in 1852. This problem remained unso!ve3d for more than a century until a computer aided proof was produced by Kenneth
Appel and Wolfgang Heken. Then a simple analytical proof was proposed twenty years later by Robertson, Seymour, Sanders and Thomas.

The use of linear algebra in algebraic graph theory, especially studies the spectrum of the adjacency matrix of a graph. By studying the adjacency matrix one can completely determines a graph and some special properties of graph.

Likewise the theory of groups particularly permutation group provides an interesting and powerful abstract approach to the study of various families of graph based on symmetry such as symmetric graphs, vertex transitive graphs, distance regular graphs, edge transitive graphs and strongly regular graphs and the inclusion relationship between these.

Several important results have been proved in the field of algebraic graph theory. The study of automorphisms of graphs is an important aspect in this field. The results obtained by Frucht (1938) opened the path of group theoretic applications in graph theory. Frucht showed that for every finite group G, there is a graph r whose automorphism group is isomorphic to G.

But the result mentioned above does not hold for permutation group. Kagno while investigating the graph of vertices < 6 and their group of automorphims, noticed the fact that given a permutation group G acting on *n* letters, there might not be a graph of *n* vertices whose automorphism group is G.

C. Y Chao gave an algorithm for constructing all the graph G with *n* vertices whose permutation group contains a given transitive permutation group of degree *n.* Debra L. Boutin defines a determining set for an automorphism group of a graph. A set of vertices S is called a determining set for a graph if every automorphism of G is uniquely determined by its action on S.

G. Chartrand, H. Gavls, and D, W. Vanderjagt (1999) develops some new idea on near automorphism of a graph. A near automorphism of a connected graph G is a permutation/of the vertices of graph that is not an automorphism but is quite close to being an automorphism in the sense that/minimizes the positive sum over all pairs of distinct vertices *{x t y }* of G, of the absolute difference of the distance between *x* and *y* and the distance between their images *f (x)* and *f (y)* . In more detail, let $d_f(x, y) = \backslash d (x,y)\sim d (f(x), f(y))|$ • Th^e^ total relative displacement of G with respect of*f* denoted by $d_f(G)$ ^{t0} be the sum of $d_f(x,y)$ overall unordered pairs *{xy y)* of distinct vertices of G. Let *n (G)* denote the smallest positive value of $d_f(G)$ among the n permutations **/** of the *n* vertices of G. A permutation *f* such that $d_f(G) = n(G)$ is called near automorphism. Charrand et al. [22] observed that *x(G)* is even and conjectured that $7t\{P_n) = 2n - 4$ where /> is a path with n vertices which was soon verified by W. Aitken (1999) by characterizing all the near automorphism of P_n . Reid K.B (2002) determined *^(J)* for all complete multipartite graphs. Recently Change et al. proved >r(C,)=4[/i/2]> where C is the graph consisting of a cycle with *n* vertices and determined all near automorphisms C_H of.

The aspect to connect graph theory with rings or other algebraic properties are another topic of research. The notion of zero divisor graph for a commutative ring R was first introduced by Beck [13]. The zero divisor graph was defined by considering all the elements of a commutative ring *R* as the vertex set of the graph and any two different elements of a commitative ring R as the vertex set of the graph and any two different elements *x* and *y* are adjacent if and only if *xy* = 0. In his introductory paper, Beck was mainly interested in presenting the concept of coloring of commutative rings by emphasizing the combinatorial properties of the zero divisors of a commutative ring. The equality of chromatic number and the clique of R for certain classes of rings like reduced and principal ideal rings was also established by Beck.

A strong countcr example to one of Beck's conjectures was produced by D.D Anderson and M. Naseer. Anderson and Naseer also established the fact that for a finite local ring the equality of *%{R)* and clique (R) does not hold under certain conditions. However by continuing with Beck's investigation of coloring of rings some positive findings for finite were obtained by Anderson and Naseer. In 1999 D.F Anderson and Philip Livingston modified the definition of zero divisor graph fi® defined by Beck by considering the vertex set as the non-zero zero divisors of a commutative ring R and provided some interesting nature of zero divisor graphs. The main concern of this paper is to study the interplay of ring-theoretic properties of a commutative ring R with graph-theoretic properties of *T (R)* by showing that the zero divisor graph is always connected with diameter less than or equal to three. They also determined which complete graph and star graphs may be realized as the zero divisor graphs T(i?) . Because of the lack of the closure law under addition, the zero divisors Z(R) of a ring R does not form an ideal within a ring. It is of interest of study the condition when Z(R) does form an ideal. Axtell [10] determined the complete classification of the sets of zero divisors that form ideals based on their zero divisor graphs.

References :

- Aitken W., Total relative displacement of permutations. J Comb Theory Ser A 87:121.
- Afkhami M. Khashyarmanesh K., The cozero-divisor graph of a commutative ring, Southeast As Bull. Math. Vol. 35(2011), 753-762.
- Akbari, S., Maimani, H.R., Yassemi, S. (2003). When a zero-divisor graph

is planar or a compled partite graph. J. Algebra, 270:169-180.

> Akhtar R., Lee L., Connectivity of the zero-divisor graph for *Jhi\c* rings, to appear in Journal Commutative Algebra.

> Anderson D. **F.,** Badawi A., On the Zero-Divisor Graph of A Ring Communications in Algebra, \ 36, Pp. 3073-3092,2008