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# Study of Littlewood's Three Principles, Egoroff's Theorem, and Lusin's Theorem

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**Abstract :** Three Principles after John Edensor Littlewood (1885–1977). Quoting from the Saint AndrewsMath HistoryWebpage: "Almost all of Littlewood's mathematical research was in classical analysis, but in this area he looked at a remarkable range of subjects and he used an even broader range of techniques in proving his results. For 35 years he collaborated with G. H. Hardy working on the theory of series, the Riemann zeta function, inequalities, and the theory of functions."

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**These are known as Littlewood's**

1. Every measurable set is "nearly" a finite union of intervals.
2. Every measurable function is "nearly" continuous.
3. Every convergent sequence of measurable functions is "nearly" uniformly convergent.

1. Let $E$ be a measurable set of finite outer measure. Then for each $\varepsilon > 0$, there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^n$ for which if $\mathcal{O} = \cup_{k=1}^n I_k$, then

$$m(E \setminus \mathcal{O}) + m(\mathcal{O} \setminus E) = m(E \triangle \mathcal{O}) < \varepsilon$$

where "$\triangle$" represents the symmetric difference between two sets: $A \triangle B = (A \setminus B) \cup (B \setminus A)$. This is Theorem 2.12 with outer measure replaced by Lebesgue measure.

2. Let $f$ be a real-valued measurable function on $E$. Then for each $\varepsilon > 0$, there is a continuous function $g$ on $\mathbb{R}$ and a closed set $F$ contained in $E$ for which

$$f = g \text{ on } F \text{ and } m(E \setminus F) < \varepsilon.$$

3. Assume $E$ has finite measure. Let $\{f_n\}$ be a sequence of measurable functions on $E$ that converges pointwise on $E$ to the real-valued function $f$. Then for each $\varepsilon > 0$, there is a closed set $F$ contained in $E$ for which

$$\{f_n\} \to f \text{ uniformly on } F \text{ and } m(E \setminus F) < \varepsilon.$$

Each of Littlewood's Principles involves (roughly) behavior "except on a set of measure "." With this idea informally defined as the term "nearly," we can paraphrase Littlewood's Three Principles as:

1. Let E be a measurable set of finite outer measure. Then E is "nearly" a finite disjoint union of open intervals.
2. Let f be a real-valued measurable function on E. Then f is "nearly" continuous.
3. Let {fn} be a sequence of measurable functions on set E of finite measure that converges point wise on E to real-valued function f. Then {fn} "nearly" converges uniformly to f.

**Theorem 0.1** (Egoroff's Theorem) Let $E$ be a set of finite measure, and $\{f_n\}$ a sequence of measurable functions on $E$ such that $f_n \to f$ pointwise on $E$. Then given $\epsilon > 0$, there is a closed set $F$ with $F \subseteq E$ such that $f_n \to f$ uniformly on $F$ and $m(E - F) < \epsilon$.

**Theorem 0.2** (Lusin's Theorem) Let $f$ be a real-valued, measurable function defined on a set $E$. Then given $\epsilon > 0$ there is a function $g$ continuous on $\mathbf{R}$, and a closed set $F$ with $F \subseteq E$ such that $f = g$ on $F$ and $m(E - F) < \epsilon$.

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