Study of Properties of inverse trigonometric functions

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Abstract : Inverse of a function '*f*' exists, if the function is one-one and onto, Since trigonometric functions are many-one over their domains, we restrict their domains and co-domains in order to make them one-one and onto and then find their inverse.

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Domains And Ranges

The domains and ranges (principal value branches) of inverse trigonometric functions are given below

Functions	Domain	Range (Principal value branches)
$y = \sin^{-1}x$	$[-1,1]$	$\frac{-\pi}{2},\frac{\pi}{2}$
$y = \cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$y = \mathrm{cosec}^{-1}x$	$\mathbf{R}-(-1,1)$	$\frac{-\pi}{2},\frac{\pi}{2} - \{0\}$
$y = \sec^{-1}x$	$\mathbf{R}-(-1,1)$	$[0,\pi] - \frac{\pi}{2}$
$y = \tan^{-1}x$	$\mathbf{R}$	$\frac{-\pi}{2},\frac{\pi}{2}$
$y = \cot^{-1}x$	$\mathbf{R}$	$(0,\pi)$

The Definition of Inverse trig functions can be seen as the following formulas. Each is the inverse of their respective trigonometric function. Also, each inverse trig function also has a unique domain and range that make them one-to-one functions.

•Inverse Sine function= arcsinx

•Inverse Cosine Function= arccosx

•Inverse Tangent Function= arctanx

•Inverse Secant Function= arcsecx

•Inverse Cotangent Function= arccotx

•Inverse Cosecant Function= arccscx

Properties of inverse trigonometric functions :

1. $\sin^{-1}(\sin x) = x \quad : \quad x \quad \frac{-}{2}, \frac{}{2}$

$\cos^{-1}(\cos x) = x \quad : \quad x \quad [0, \;]$

$\tan^{-1}(\tan x) = x \quad : \quad x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1}(\cot x) = x \quad : \quad x \in (0, \pi)$

$\sec^{-1}(\sec x) = x \quad : \quad x \quad [0, \pi] - \frac{\pi}{2}$

$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad : \quad x \quad \frac{-\pi}{2}, \frac{\pi}{2} - \{0\}$

2. $\sin(\sin^{-1} x) = x \quad : \quad \in [-1, 1]$

$\cos(\cos^{-1} x) = x \quad : \quad x \in [-1, 1]$

$\tan(\tan^{-1} x) = x \quad : \quad x \in \mathbf{R}$

$\cot(\cot^{-1} x) = x \quad : \quad x \in \mathbf{R}$

$\sec(\sec^{-1} x) = x \quad : \quad x \in \mathbf{R} - (-1, 1)$

$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x \quad : \quad x \in \mathbf{R} - (-1, 1)$

3. $\sin^{-1}\dfrac{1}{x} \quad \text{cosec}^{-1}x \quad : \quad x \in \mathbf{R} - (-1,1)$

$\cos^{-1}\dfrac{1}{x} \quad \sec^{-1}x \quad : \quad x \in \mathbf{R} - (-1,1)$

$\tan^{-1}\dfrac{1}{x} \quad \cot^{-1}x \quad : \quad x > 0$

$= -\pi + \cot^{-1}x \quad : \quad x < 0$

4. $\sin^{-1}(-x) = -\sin^{-1}x \quad : \quad x \in [-1,1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x \quad : \quad x \in [-1,1]$

$\tan^{-1}(-x) = -\tan^{-1}x \quad : \quad x \in \mathbf{R}$

$\cot^{-1}(-x) = \pi - \cot^{-1}x \quad : \quad x \in \mathbf{R}$

$\sec^{-1}(-x) = \pi - \sec^{-1}x \quad : \quad x \in \mathbf{R} - (-1,1)$

$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x \quad : \quad x \in \mathbf{R} - (-1,1)$

5. $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2} \quad : \quad x \in [-1,1]$

$\tan^{-1}x + \cot^{-1}x = \dfrac{\pi}{2} \quad : \quad x \in \mathbf{R}$

$\sec^{-1}x + \text{cosec}^{-1}x = \dfrac{\pi}{2} \quad : \quad x \in \mathbf{R} - [-1,1]$

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6. $$\tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x \quad y}{1 - xy} \ : \ xy < 1$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right); xy > -1$$

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7. $2\tan^{-1}x = \sin^{-1}\frac{2x}{1 \quad x^2} \qquad : \qquad -1 \leq x \leq 1$

$$2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1 \quad x^2} \qquad : \qquad x \geq 0$$

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} \qquad : \qquad -1 < x < 1$$

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