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A computer-based approach to number theory learning : A review

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Abstract

The visual presentation of the natural numbers within *Number Worlds* offers an alternate representation of the numbers compared to that of the more traditional number line. This alternate representation accentuates the patterns inherent in arithmetic sequences of multiples, as well as arithmetic sequences of non-multiples generated by 'shifting' the multiples by an integer value. It also allows users to see patterns, or lack thereof, in the structure of factors, square numbers, and primes. The second feature influencing the participants' understanding of basic elementary number theory relationships lies in the ease with which the microworld accommodates experimentation. Participants were able to interact with *Number Worlds* in a way that allowed them the freedom to think both *about* the microworld and *with* it. These two features – visual representation and experimentation – are in fact closely interrelated. The crucial link between the two is the attention to patterns. Number patterns presented in a novel visual manner provoked aesthetic responses that invited experimentation, which generated new patterns, thereby reinforcing aesthetic engagement. Furthermore, the regularity of visual patterns led students to make conjectures, which could be in turn tested through experimentation and visual feedback. The participants' conjectures show their perceptions of the relationships among the concepts of number theory, and the depiction of these concepts in the microworld.

Introduction.

Over the last few years a number of different security mechanisms have been developed in order to protect users from different kinds of attacks eg. the SSL/TLS protocol. Some of these mechanisms have been formally proven to be secure and evaluated based on international security standards such as the Common Criteria or ISO 27001. However, a number of user studies , as well as the prevalence of attacks successfully targeting the human end-user, demonstrate that many of these security mechanisms falter and fail as soon as the user is involved in the process. One big problem is the large number of security warnings user smare confronted with. Users are habituated into ignoring these since they do not understand and thus perceive any risk . This can either be attributed to the 'stupidity' of the user (not understanding what is secure and what is not) or the 'obtuseness' of the developers (not designing systems properly and not giving due consideration to the non-security related nature of the end-user's primary goal or task). As a solution, one could try to eliminate the end-user from the security mechanism's operation altogether.

A computer-based approach to number theory learning

To satisfy both our research aims – to better understand some of the learning difficulties summarised above – and our teaching objectives – to provide students with more powerful ways of learning – one author (Sinclair) designed a computer-based learning environment focusing on the domain of elementary number theory. We first outline our rationale for developing this environment, a number theory microworld. Then we describe the design principles we followed, and their connection to
the research summarised above. Finally, we present an analysis of the mathematical concepts and behaviours supported by the microworld.

Affordances of Computer-based Learning Environments

Many researchers, such as, have argued that well designed computer-based learning environments can provide a scaffold for reasoning by fostering the development and use of visual and experimental reasoning styles, which greatly complement the traditionally-taught symbolic-deductive methods. For the learning of elementary number theory, few, if any, such computer-based environments exist. In addition, little is known about the kinds of visual representations and experimental interfaces that might help students learn and understand the wide range of number theory-related ideas involved in the research summarised above. Certain mathematical subjects, for example fractal geometry, easily lend themselves to a visual and experimental form of mathematical inquiry, but is the same necessarily true of number theory? We were aware that some professional number theorists take advantage of the visual and experimental approaches afforded by the computer; however, we wanted to establish whether such approaches are possible and appropriate for student learning. Factor trees and number lines provide one type of visual representation frequently encountered in the sub-domain of elementary number theory. Alongside such visual tools, push-button calculators can support a certain range of numeric experimentation. However, these tools suffer from a degree of isolation: for example, neither factor trees nor number lines can be represented on calculators. In contrast to an approach based on collections of tools, we were interested in the class of learning environments called "microworlds" which, as describes, "embody" or "instantiate" some sub-domain of mathematics. The *microworld* is intended to be a mini-domain of mathematics that offers an external representation of a (sub)set of mathematical ideas, and brings tools together into a phenomenological whole. The challenge for mathematics educators is to design microworlds that can offer new external systems of representation that foster more effective learning and problem solving.

Mathematical Ideas Inherent in Number Worlds

Elementary number theory is concerned with the structures and relationships of natural numbers. Therefore, in designing the *Number Worlds* microworld, we have chosen to focus primarily on this set of numbers. However, instead of the one-dimensional number line representation that is traditionally used, we adopted a two-dimensional grid display, thus maximising the use of 'real estate' on the screen. The two-dimensional grid display also sheds a different light on the relationships among the numbers and provides an opportunity to construct or reconstruct these relationships. In addition, it offers a different external representation of both factors and multiples, as well as primes and square numbers, in ways that provide concrete visual instantiations of algebraic relationships. Further, by producing unexpected patterns, the two-dimensional grid display offers a novel representation of the numbers, which we hoped would provoke surprise and engagement for learners. In what follows we provide several examples of the mathematical relationships inherent in *Number Worlds*.

These tasks were intended to familiarise the participants with all the aspects of *Number Worlds* and to create an environment of experimentation and conjecturing. To assist the participants, computer labs were scheduled at different times on different days. Participation in the lab was optional; never the less about one-half of the participants chose to work, for at least part of the assignment, in this environment. Our data consists of three main sources: observations of participants' work during the lab time, written assignments and clinical interviews:

1. During the allocated lab hours, the work of the participants was observed and assistance given where necessary. We noted their frequently asked questions, chosen routes for exploration, conjectures, as well as their approaches toward testing conjectures. We also used these observations as a guideline for designing the interview questions.

2. Following the tasks for exploration, participants were provided with an additional list of tasks, for which a written response was requested. Participants had two weeks to complete the assignment. The list of tasks can be found in Appendix 2. These tasks specifically request that the participants make an observation and, whenever possible, generalise it. For example:

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Describe at least three different ways for creating a grid of diagonals. Describe a general procedure for creating a grid of diagonals. Explanations were expected, even when not explicitly requested, as a routine practice in the course.

3. After the completion of the written assignment, clinical interviews were conducted with seventeen volunteers from the group. The interviews lasted 30–50 minutes; the *Number Worlds* microworld was available to the participants at all times during the interview. These interviews were semi-structured, that is, the questions were designed in advance , but the interviewer had the liberty to follow up with prompts, include additional questions, or omit questions due to time considerations.

Conclusions

In this article we have outlined the design features of the *Number Worlds* microworld and described the interaction of a group of pre-service elementary teachers with this microworld. Prior research has repeatedly shown that the concepts and relationships underlying elementary number theory are elusive and fleeting for students. In comparing the work of participants in this study with that research, we find that interaction with *Number Worlds* had a positive effect in helping participants construct these concepts. In particular, the previously reported confusion between factors and multiples, belief in larger numbers having more factors, and the lack of recognising the 'every nth' property of multiples were either absent or infrequent among the participants working with *Number Worlds*. Furthermore, we observed stronger understanding of factors and multiples, especially in relation to their distribution within the natural numbers. Based on observations in the computer lab, students' written assignments and clinical interviews, we attribute this effect to the two major factors we had hypothesised would be pivotal: (1) the novel visual representation inherent in *Number Worlds* and (2) the possibility of experimentation.

REFERENCES

1. Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning* 7(3): 245–274.
2. Bailey, D., Borwein, J. and Girgensohn, R. (to appear). *Experimental Mathematics: Computational Paths to Discovery*. Boston, MA: AK Peters.
3. Brown, S. (1978). *Some "Prime" Comparisons*. Reston, Virginia: National Council of Teachers of Mathematics.

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4. Brown, A., Thomas, K. and Tolias, G. (2002). Conceptions of divisibility: Success and understanding. In S.R. Campbell and R. Zazkis (Eds), *Learning and Teaching Number Theory: Research in Cognition and Instruction* (pp. 41–82). Westport, CT: Ablex.
5. Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics? *Educational Studies in Mathematics* 37(2): 121–143.
6. Devlin, K. (2000). *The Math Gene: How Mathematical Thinking Evolved and Why Numbers Are Like Gossip.* New York: Basic Books..