



A STUDY ON THE CONCEPT, THEORY, DEVELOPMENT PROCESS AND APPLICATIONS OF NUMBER THEORY

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ABSTRACT

This paper basically analyzed the growth and application of several theories trying to review the history of this discipline, and discover its impact on the creation and our life and its basic application. Several hypothesis is belonged to its originally to the analysis of the integers. With the number of contributors made by several mathematicians in various ages to modifying the analysis of the integers, the basic system if the number hypothesis has been slowly enhanced and thereby an entire and unified discipline has been produced. As a fundamental discipline, several hypotheses imparts profound makes an impact on the various disciplines and also the basic fundamental foundation of several disciplines.

Keywords: Number theory, concept, theory, development process, application

I.INTRODUCTION

As we know, Number theory which is the branch of the mathematics is also known as the queen of mathematics that basically distress about the positive integers 1, 2,3,4,5 which are mostly known as the natural numbers and their ultimate characteristics and properties. From the ancient times, all these natural numbers classified as odd numbers, even numbers, square numbers, prime numbers, Fibonacci numbers, triangular numbers, etc. Because of lots of unsolved problems, here number theory plays an essential part in the field of mathematics. The earlier explanation of number theory relying on the tools used to solve the related issues and problems which are shown in the figure below:

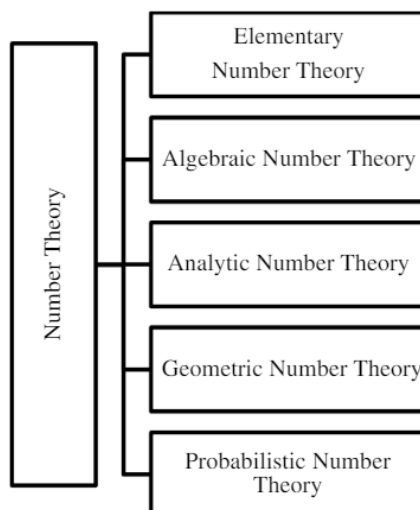


Figure: Modern classification on Number Theory

Basically the analysis on the integers in a scientific way is actually recognized to the Greeks. Well ahead, a big revolution ion this hypothesis occurred because of the advent of the popular book “elements” by Euclid in which the mathematics itself is portrayed with the exact and accurate evidences.



A. The Concept of Number Theory

Just as per the name depicts, Number theory is a hypothesis concentrating on number. Greater than 3000 years ago, the basic context of number and arithmetic has happened. This term “number theory” was arithmetic in the ancient era and also was succeeded by “number theory “in the recent 20th century. Number theory is include in the branch of the mathematics where mathematics refers to the foundation of several sciences and engineering disciplines though the number theory is the foundation of mathematics. Number theory initially mainly deals with8 the nature of integers. Most of the questions in field of number theory are short and the prime way to solve all these questions is special components decomposition. Additionally, in the methods of reconstructing the special factorization, most of the modern basic concepts like complex integers, fundamental numbers are created which also give modern analysis techniques for number theory solution [16].

B. The Subdivisions of Number Theory

In the mathematical field, the number theory can be classified into several classes of which elementary number theory, analytic number theory, algebraic number theory and geometric number theory are more essential one. It also comprised of some famous number theory classes like combinatorial number theory and transcendental number theory. The main concentrations of these sub classes and their dissimilarities are representing in the following table:

Table: The focuses of these subdivisions and their differences

Subdivision	Explanation
Elementary number theory	Elementary number theory is a branch of number theory based on elementary method. In essence, it applies divisible property to mainly study divisible theory and congruence theory. The typical conclusions in this theory include the familiar congruence theorem, Euler's theorem, and Chinese residual theorem and so on.
Analytic number theory	Analytic number theory studies the integers with calculus and complex analysis. Some analytic functions, such as the Riemann function ζ which studies the properties of integers and primes, can also be employed to understand number theory.
Algebraic number theory	Algebraic number theory is more inclined to study the nature of various rings of integers from the perspective of algebraic structure.
Geometric number theory	Geometric number theory studies the distribution of the integers from the perspective of geometry.
Computational number theory	Computational number theory studies questions in number theory with computer algorithms.

C. The Significance of Number Theory

For a long time, number theory basically explained the fundamental characteristic of mathematics, so it was characterized as a discipline without the application value. With the deep and thoughtful scientific and high tech transformation brought by the innovation and growth of computer, number theory has been highly used, and is formerly just a pure and clean mathematics, but a mathematical discipline with applied application value. Currently, number theory is highly and broadly used and completely applied



in several fields of education like physics, biology, chemistry, communication, graphics, computing, musicology and cryptography[20].

This also shows that the importance of numbers theory that it can be highly and completely applied to several fields comprising mathematics, and has grown into a modern practical mathematics disciplines-practical number theory. Thus, number theory is formerly just a pure and complete discipline of mathematics but an authentic practical discipline. Arbitrated from the present theory pattern and techniques of number theory, this old discipline is surrounded to be a dynamic.

II. THE DEVELOPMENT OF NUMBER THEORY

A. *The Development of Number Theory and Algebra*

Several queries in number theory have been stated and then solved which appeals large number of peoples to concentrates on the number theory. In the extensive part of history, several types of process and methods are introduced to solve problems. Algebraic number theory has been enhanced with the growth of various field sand practical applications. The popular philosopher, Bacon said that past makes people smart and innovative therefore it is important to discover the growth of recent algebraic number theory. Several local analyses on the algebraic number theory are basically complete discussion on the enhancements of algebraic number theory. On the basis of collection and integration of pertinent information, this paper is basically dedicated to investigating the birth of algebraic theory by studying the prime issues in the growth of two higher reciprocity laws and Theorem of Fermat. With the evolvment of modern thoughts to recognize the history, this paper attempts to make more broad and wide study and thoughtful thinking.

a) *The phase of Arithmetic*

In the course of an era from around 3,800 to the third century, arithmetic images were not same, and algebra was completely separated from geometry. The prehistoric Greeks made the greatest commitment to number theory, including some famous accomplishments, for example, Euclid's Euclidean algorithm in geometry which recommended that the number of prime numbers is infinite, and the principal theorem of arithmetic which was associated with elementary number theory fundamentally.

b) *The entire phase of number and equation theory*

In the course of the period from the seventh century to the sixteenth century, irrational and imaginary numbers were discovered.

i. The innovation of irrational numbers

Hipparsos of the Pythagorean School introduced the main irrational number, stunning the leaders of the school around then. He also suggested that everything numbers could be communicated as the ratio of integers, which prompted the primary mathematical calamity.

ii. Creation of arithmetic operators and answer for irrational equations

In India, the mathematician Brahmagupta presented a gathering of images used to communicate ideas and portray operations in the seventh century, and Posgallo later set forward the idea of negative square root, the answer for irrational equations and the calculation of irrational numbers in the twelfth century, which cultivated the investigation of algebra to another stage.

iii. Establishment of imaginary number theory



In the book *The Great Art* published in 1545 by the Milanese scholar Cardano (1501-1556), the overall answer for the cubic equation was uncovered, which was known as Cardano's recipe later. Cardano was the main mathematician to figure the square root of a negative number.

c) *The phase of linear algebra*

During the period from the seventeenth century to the nineteenth century, the apparatuses for tackling linear issues, matrices, determinants, and vectors arose, which offered types of assistance to the modern culture.

d) *The phase of abstract algebra*

During the period from the nineteenth century to the present, the significance of structure and procedure to the algebra structure was featured, which offered administrations to the data society.

III. THE CLASSICAL QUESTIONS AND CONJECTURES IN NUMBER THEORY

A. *Mersenne Prime*

Mersenne primes are gotten from Mersenne numbers which allude to the positive integers of the structure $2^p - 1$, where if the example p is prime, p is normally characterized as M_p . On the off chance that a Mersenne number is prime, it is known as a Mersenne prime; else it is known as a Mersenne number.

Prime numbers, otherwise called primes, allude to the numbers which are divisible simply by 1 and themselves, for example, 2, 3, 5, etc. Euclid has demonstrated that the number of primes is infinite with confirmations by logical inconsistency.

In the infinite sequence $2^n - 1$, Mersenne numbers and Mersenne primes just account for a little proportion, however Mersenne primes are infinite.

In the event that the type n is prime, M_n is a prime number. Nonetheless, when n is prime, M_p may not be prime (for instance, $M_2 = 2^2 - 1 = 3$ and $M_3 = 2^3 - 1 = 7$ are prime, while $M_{11} = 2^{11} - 1 = 2047 = 23 \cdot 89$ is anything but a prime number). Until further notice, 51 prime numbers have been discovered, of which the biggest one is $M_{82589933}$ with 24862048 digits. These days, dispersed organization figuring innovation has become the most recent technique to find primes [17].

B. *Goldbach Conjecture*

Goldbach conjecture is one of the most established unsolved issues in number theory. It expressed that each even whole number greater than two can be composed as the amount of two primes. Goldbach conjecture is related with whole number partition which is proposed by European number theorists around then and zeroed in on the inquiry - "would you be able to dissect integers as the amount of specific numbers with specific properties?" To be explicit, the inquiry is whether you can isolate all integers into the amount of a few complete squares, or the amount of a few complete blocks. Such a partition of a given considerably number into the amount of two prime numbers is called Goldbach examination. Goldbach conjecture set aside a long effort to create. China's mathematician Chen Jingrun demonstrated that each adequately enormous much number can be composed as the amount of some prime number and another number which is the product of two primes [15].

In light of Goldbach conjecture of even numbers, the conjecture that each odd integers greater than 7 can be composed as the amount of three primes has been proposed, which is known as the powerless Goldbach conjecture. It has been demonstrated in 2013.



C. *Fibonacci Sequence*

Fibonacci sequence, characterized by Italian mathematician Leonardo Fibonacci, alludes to a progression of numbers in which starting from the third number in the sequence; each number is the amount of the two going before ones. The n th number in the sequence can be signified by $f(n)$, and its recursive sequence can be communicated as the accompanying recipe.

$$f(n) = f(n - 1) + f(n - 2)$$

Uses of Fibonacci sequence:

- 1) Golden ratio: as the number of things in the sequence builds, the ratio of the previous to the last expands nearer to the golden ratio.
- 2) Pascal triangle: the numbers on diagonals of Pascal triangle add to the Fibonacci sequence.
- 3) Area of a rectangle shape: the squares of the initial not many numbers in the Fibonacci sequence are treated as various little quadrilateral territories, and they can be consolidated into huge quadrilateral regions.

D. *The Significance of Mathematical Conjectures*

Besides from the given conjectures, there are numerous different types of conjectures. The greater part of the mathematical conjectures depends on perception, confirmation, acceptance and speculation of a huge number of realities. Such a strategy for abstracting the general and basic properties from the extraordinary properties is a significant driver of mathematical exploration. The articulation and examination of mathematical conjectures distinctively mirror the use of persuasions in science. Besides, mathematical conjectures advance the investigation of mathematical technique[5].

Moreover, mathematical conjectures regularly assume the part as the significant marker of mathematical turn of events. Fermat conjecture brought forth algebraic number theory, while Goldbach conjecture advanced the improvement of screening strategies. Riemann conjecture demonstrated the prime number theorem, while the Four-shading conjecture was tackled by PC, and accordingly another time of machine check has been opened. In this way, mathematical conjectures are the valuable gemstones, yet additionally the vital driver in the advancement of science[3].

IV. APPLICATIONS OF NUMBER THEORY

In the ancient period, a branch of pure and clear mathematics was initially less implemented in reality. But after the integration with the current computation methods, it also gives answers to several problems that evolved.

Cryptography is basically refers to the one of the essential fields in current period of digital era, where online privacy is a big distress. Whenever a message that has been sent from a sender to receiver during online communication and interaction, there was a risk of being seen by some unknown persons with proper privacy and safety. Further, this issues or concerns have been solved by the introduction of decryption and encryption. Encryption basically refers to that large number which is encoded to the message which is sent by the sender, it is also said to be a 'key'; and to decode that message, and the receiver should also have the same key. The uses of number theory is in the generation of some large number i.e. usually prime. As per the theory of Maurer, a productive algorithm to produce such numbers with the assistance of the number theory. Congruence modulo relation is considered as a part of the modular arithmetic, a basic part of the number theory. It is integrate acoupled with the linear transformations which plays a significant role in cryptography. A natural number is basically stated as the product of two prime numbers i.e. a_1 and a_2 where a_1 and a_2 doesn't require to be distinct is called semi



prime. Semi primes are remarkable very helpful in the part of cryptography, most special and unique in public key cryptography[9].

‘Elliptic Curves’ is an essential concept in field of number theory. Large number of investigation on the number theories sequence regarding to the elliptic curves was formerly followed primarily for imaginative details. Earlier, all these queries have emerges crucial in several applied regions comprising of theory of coding, number generation and primarily cryptography [4]. As per the discussion of Srikanth, the super ultimate elliptic Diophantine equation which is a vital part of the field of the number theory which is engaged for several application which is based in the computer coding.

There are several surprising series and sequences of numbers which have utmost essential for problem solving. One of the series is Fibonacci series i.e. 0, 1, 1, 2, 3, 5, 8..... It has also various applications which are useful in field of engineering. In computer science engineering, the techniques of ‘Fibonacci search technique’ is basically discussed by Ferguson, is a method of finding an easy array. It basically divides and overcomes algorithm. This calculation basically assists to narrow down the basic locations of the needed element with the assistance with of Fibonacci numbers. One of the essential conveniences of Fibonacci series that one only requires to perform addition and subtraction to evaluate the indices of the accessed array components rather than various deady operations[6].

Many theorems are very useful in field of mathematics but one of the popular theorems in mathematics is Pythagoras theorem. It basically deals with the right angled triangles and gives the relation between the sides. Some examples are set here for better understanding of the theorems. The wing configuration utilized in modern jet aircraft is the ‘Delta wing’ dedicated to the right angled triangle as this plays an efficient and productive design of these configurations. Some Aerospace scientist and meteorologist determine a range and source of sound by using this Pythagoras theorem. One of the interesting combinations of non-arithmetic sequence and the Pythagoras theorem exists in the number theory. This series 3, 5, 9, 11, 15, 19, 21, 25, 29, 35, consists of legs as odd numbers in right triangles with the length of the sides being integers and hypotenuse length as a prime number [9].

A. Cryptography

With the entire growth of the technology of network encryption, number theory has determined its own position cryptography. A few years ago, a professor Wang Xiaoyun cracked the MD5 code from the help of number theory in the School of Shandong University. There is some indefinite and unusual appearance of prime factors in composite numbers, it is quite difficult to decode the composite number into the product of any prime numbers and at the same time, it is very challenging to enlighten peoples to use it to propose difficult codes.

At the time of studying number theory particularly cryptography, we need to follow deterministic algorithm and set of rules instead than probabilistic rules, and we will only lower our needs and implement probabilistic algorithm is there is none of the chances of deterministic algorithm.

B. Computer Animation

Most of the time linear transformation is generally to make pictures and computer graphics are to build graphics on showing devices by programs and algorithms, therefore the technology of the linear transformation can be used to make animation in computer. Graphics of computer basically consist of showing8 of images, calculation and storage. With eth enhancements in the potential of software, the technology of linear transformation is basically used in animation of computer.



C. *Machine Translation*

One of the main algorithm or calculation of machine translations is basically based on the statistical method, with near about 90% precision. Moreover, this calculation is also utilized in technology of searching of image. The main principle of this technology is that the language units of source language and target language can be exhibited by vectors, and the lexical vectors of various languages can be estimated onto a two dimensional plan for study. Results of experiments exhibit that the lexical vectors of various languages do have some connection as same as to linear connection, therefore, it is of important to determine machine translation as a linear transformation.

D. *Other Basic Fields*

Moist of the time, number theory plays a shocking roe in otherhypothesis. In quantum theory, Hermite operator is one of the most fundamental theories. With eh exception from that, number theory is also broadly utilized in non-mathematical disciplines, like information sciences, quantum chemistry and theoretical physics etc.

V.CONCLUSION

In the entire paper, we basically discussed about the basic concept and theories, growth techniques and uses of number theory. As a fundamental sciences and engineering disciplines, the growth pattern and standard of mathematics has a philosophical impact on various disciplines. BY analyzing the growth of number theory and its application, the objective if this review is to help readers to acquire the growth an existence of number theory and its upcoming trend in the integration if computer science. Currently, with the fast growth of computer field, number theory or even several mathematical disciplines will make higher advances and enhancement in the future time.

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