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ROUGH PRIME BI - IDEAL IN SEMIRINGS

Dr. V.S.Subha
Assistant Professor, Department of Mathematics
Annamalai Unversity, Annamalainagar-608002, India, e-mail: surandsub@yahoo.com.

Abstract: In this paper we introduce the notions of rough prime bi-ideals, strongly rough prime bi-ideals and rough irreducible bi-ideals of semirings. We have to show that the lower and upper approximations of a prime bi-ideal are also prime bi-ideals

Index terms: Prime bi-ideals Strongly rough prime bi-ideals rough irreducible bi-ideals Rough semi prime ideals, Strongly rough irreducible bi-ideals, rough irreducible bi-ideals.

1.INTRODUCTION

Semiring which are common generalization of also relative ring and distributive lattice are found in abundance around us Vandiver[22] introduced semirings. Iseki[6] introduced the notion of ideals in semirings. Shabi and Kanwal[16] introduced prime bi-ideals in semigroups. Bashir et.al.,[1] introduced prime bi-ideals in semirings.

The notion of rough sets was introduced by Pawlak in his papers [11-14]. Rough set theory is an extension of set theory, in which a subset of a universe is described by a pair of ordinary sets called the lower and upper approximations. Rough sets are a suitable mathematical model of vague concepts, i.e., concepts without sharp boundaries. It soon invoked a natural question concerning possible connection between rough sets and algebraic systems. The application of rough set theory in the algebraic structure was studied by many others such as Z.Bonikowaski[3], J.Pomykala[15], Y.B.Jun[8], T.Iwinski[7]. The notion of rough ideals was introduced by N.Kuroki[9]. Biswas and Nanda[2] introduced rough groups and subgroups.

R.Chinram[4] studied Rough prime ideals in Γ–semigroups. Thillaigovindan and V.S.Subha [20,21] introduced rough prime bi-ideals in Γ–semigroups. V.S. Subha [17-19] introduced rough k-ideal and quasi-ideals in semirings. K.Osaman and B.Davvaz[5,10] discused rough ideals in rings.

2. PRELIMINARIES

In this section we reproduce some basic concept which are needed in the sequel. A semiring is a non-empty set R together with two binary operations additions '+' and multiplication '.' Such that $(R,+)$ is a commutative semigroup and $(R,\cdot)$ is a semigroup where two operations are connected by ring like distributive laws, that is $a(b+c) = ab + ac$ and $(*)$

Let U be a universal set. For an equivalence relation ρ on U, the st of elements of U that are related to $x \in U$, is called the equivalence class of x and is denoted by $[x]$. Let U/ρ denote the family of equivalence classes induced by ρ on U. U/ρ be a partion of U such that each element of U is contained inexactly one equivalence class. A semiring R is called commutative semiring if multiplication is commutative. A nonempty subset B of a semiring R is called a subsemiring of R if for all $a, b \epsilon B$, we have $a + b \epsilon B$ and $ab \epsilon B$.

A *left(resp. right) ideal* I of a semiring R is a nonempty subset of R such that $a + b \epsilon I$ for all $a, b \epsilon I$ and $xa \epsilon I$ (resp. $ax \epsilon I$) for all $a \epsilon I$ and $x \epsilon R$.

An *ideal* of a semiring R is a subset of R which is both a left ideal and right ideal of R. A non empty subset Q be a quasi- ideal of R , we mean a subsemigroup Q of R such that
$RQ \cap QR \subseteq Q$.

A nonempty B of a semiring R is called *bi-ideal* of R if B is a subsemiring of R and $BRB \subseteq B$.

A semiring R is called *Von Neumann regular* or *simply regular* if for each $a \epsilon R$ there exists $x \epsilon R$ such that $axa = a$.

A semiring R is called an *intra-regular* semiring if for each $a \epsilon R$ there exist $x, y \epsilon R$ such that $a = \sum_{i=1}^{n} x_i a^2 y_i$.

Definition 2.1.[17] Let θ be an equivalence relation on R. θ is called a *congruence relation* if $(a, b) \in \theta$ implies

(i) $(a + x, b + x) \in \theta$; (ii) $(x + a, x + b) \in \theta$; (iii) $(ax, bx) \in \theta$ and (iv) $(xa, xb) \in \theta$, for all $x \in R$.

The following theorem is an immediately consequence of Definition 2.1.

Theorem 2.2.[17] *Let θ be a congruence relation on a semiring R.* Then $(a, b), (c, d) \in \theta$ implies $(a + c, b + d) \in \theta$, $(ac, bd) \in \theta$ for all $a, b, c, d \in R$.

Lemma 2.3. *Let θ be a congruence relation on R. If $a, b \in R$, then*

(i) $[a]_\theta + [b]_\theta \subseteq [a + b]_\theta$

(ii) $[a]_\theta . [b]_\theta \subseteq [ab]_\theta$.

A congruence relation θ on R is called complete if $[a]_\theta + [b]_\theta = [a + b]_\theta$ and $[a]_\theta . [b]_\theta = [ab]_\theta$.

Theorem 2.4. [17] *Let θ and ψ be congruence relations on R and let A and B be nonempty subsets of R. Then*

(i) $\underline{\theta}(\mathrm{A}) \subseteq \mathrm{A} \subseteq \overline{\theta}(\mathrm{A})$

(ii) $\underline{\theta}(\emptyset) = \emptyset = \overline{\theta}(\emptyset)$

(iii) $\underline{\theta}(\mathrm{R}) = \mathrm{R} = \overline{\theta}(\mathrm{R})$

(iv) $\overline{\theta}(\mathrm{A} \cup \mathrm{B}) = \overline{\theta}(\mathrm{A}) \cup \overline{\theta}(\mathrm{B})$

(v) $\underline{\theta}(\mathrm{A} \cap \mathrm{B}) = \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\mathrm{B})$

(vi) $A \subseteq B$ implies $\underline{\theta}(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{B})$ and $\overline{\theta}(\mathrm{A}) \subseteq \overline{\theta}(\mathrm{B})$

(vii) $\underline{\theta}(\mathrm{A} \cup \mathrm{B}) \supseteq \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\mathrm{B})$

(viii) $\overline{\theta}(\mathrm{A} \cap \mathrm{B}) \subseteq \overline{\theta}(\mathrm{A}) \cap \overline{\theta}(\mathrm{B})$

(ix) $\theta \subseteq \psi$ implies $\underline{\psi}(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{A})$ and $\overline{\theta}(\mathrm{A}) \subseteq \overline{\psi}(\mathrm{A})$

(x) $(\overline{\theta \cap \psi})(\mathrm{A}) = \overline{\theta}(\mathrm{A}) \cap \overline{\psi}(\mathrm{A})$

(xi) $(\underline{\theta \cap \psi})(\mathrm{A}) \subseteq \underline{\theta}(\mathrm{A}) \cap \underline{\theta}(\psi)$

(xii) $\underline{\theta}\left(\underline{\theta}(\mathrm{A})\right) = \underline{\theta}(\mathrm{A})$

(xiii) $\overline{\theta}\left(\overline{\theta}(\mathrm{A})\right) = \overline{\theta}(\mathrm{A})$

(xiv) $\overline{\theta}\left(\underline{\theta}(\mathrm{A})\right) = \underline{\theta}(\mathrm{A})$

(xv) $\underline{\theta}\left(\overline{\theta}(\mathrm{A})\right) = \overline{\theta}(\mathrm{A})$.

Definition2.5.[1] A bi-ideal B of R is called a *prime bi-ideal* of R if $B_1B_2 \epsilon B$ implies $B_1 \subseteq B$ or $B_2 \subseteq \mathrm{B}$ for any bi-ideals B_1, B_2 of R.

Definition2.6. [1]A bi-ideal B of R is called *strongly prime bi-ideal* of R if $B_1B_2 \cap B_2B_1 \subseteq B$ implies $B_1 \subseteq B$ or $B_2 \subseteq B$ for any bi-ideals B_1, B_2 of R.

Definition2.7. [1] A bi-ideal R of R is called *semiprime bi-ideal* of R if $B^2 \subseteq B$ implies $B_1 \subseteq B$ for any bi-ideals $B_1$of R.

Obviously every strongly prime bi-ideal of a semiring is prime bi-ideal and every prime bi-ideal is semiprime bi-ideal but the converse is not true. The intersection of any family of prime bi-ideal of a semiring is semiprime bi-ideal of R.

Definition2.8. [1] A bi-ideal B of R is called *irreducible bi-ideal* of R if $B_1 \cap B_2 = B$ implies either $B_1 = B$ or $B_2 = B$ for any bi-ideal B_1, B_2 of R.

Definition 2.9. [1] A bi-ideal B of R is called *Strongly irreducible bi-ideal* of R if $B_1 \cap B_2 = B$ implies either $B_1 \subseteq B$ or $B_2 \subseteq B$ for any bi-ideal B_1, B_2 of R.

Every strongly irreducible bi-ideal of a irreducible is prime bi-ideal

3.MAIN RESULTS

In this section we introduce rough prime bi-ideals, rough strongly prime. Bi-ideals and rough semiprime bi-idelas in semirings. Throughout paper R denoted unless otherwise mentioned the semiring

Definition 3.1 Let ρ be a congruence relation on R. A bi-ideal B of R is called *rough prime bi-ideal of* R if $\bar{\rho}(B)$ and $\underline{\rho}(B)$ are prime bi-ideals of R.

A bi-ideal B of R is called *strongly rough prime bi-ideal* of R if $\bar{\rho}(B)$ and $\underline{\rho}(B)$ are strongly prime bi-ideal of R.

A bi-ideal B of R is called *rough semi prime bi-ideal* of R if $\bar{\rho}(B)$ and $\underline{\rho}(B)$ are semi prime bi-ideal of R.

Definition 3.2 .Let ρ be a congruence relation on R. A bi-ideal B of R is called *rough irreducible bi-ideal* of R if $\bar{\rho}(B)$ and $\underline{\rho}(B)$ are irreducible bi-ideal of R.

A bi-ideal B of R is called *strongly rough irreducible bi-ideal* R if $\bar{\rho}(B)$ and $\underline{\rho}(B)$ are strongly rough irreducible bi-ideals of R.

Theorem 3.3 Let ρ be a congruence relation on R. If B is a bi-ideal of R then

(i) $\bar{\rho}(B)$ is a bi-ideal of R.
(ii) $\underline{\rho}(B)$ is a bi-ideal of R.

Proof: Let B be a bi-ideal of R, then $BRB \subseteq B$.

(i)We have

$$\begin{aligned}\bar{\rho}(B)R\bar{\rho}(B) &= \bar{\rho}(B)\bar{\rho}(R)\bar{\rho}(B)\\ &= \bar{\rho}(BRB)\\ &\subseteq \bar{\rho}(B)\text{ , since } B \text{ is a bi-ideal of } R.\end{aligned}$$

Hence $\bar{\rho}(B)\mathrm{R}\bar{\rho}(B) \subseteq \bar{\rho}(B)$

Therefore $\bar{\rho}(B)$ be a bi-ideal of R.

(ii) We have

$$\begin{aligned}\underline{\rho}(B)R\underline{\rho}(B) &= \underline{\rho}(B)\underline{\rho}(R)\underline{\rho}(B)\\ &= \underline{\rho}(BRB)\\ &\subseteq \underline{\rho}(B)\text{ , since } B \text{ is a bi-ideal of } R.\end{aligned}$$

Hence $\underline{\rho}(B)R\underline{\rho}(B) \subseteq \underline{\rho}(B)$

Therefore $\underline{\rho}(B)$ be a bi-ideal of R.

Theorem 3.4. Let ρ be a congruence relation on R. If R is a prime bi-ideal of R then

(iii) $\bar{\rho}(P)$ is a prime bi-ideal of R.
(iv) $\underline{\rho}(P)$ is a prime bi-ideal of R.

Proof. Let P be a prime bi-ideal of R. Then $P_1P_2 \subseteq P$ implies that either $P_1 \subseteq P$ or $P_2 \subseteq P$ for any bi-ideal P_1 and P_2 of R. Since P bae bi-ideal of R. By Theorem [ ] $\bar{\rho}(P)$ is a bi-ideal of R. Assume that $\bar{\rho}(P_1)\overline{\rho}(P_2) \subseteq \bar{\rho}(P)$, $\bar{\rho}(P_1) \nsubseteq \bar{\rho}(P)$ and $\bar{\rho}(P_2) \nsubseteq \bar{\rho}(P)$. Since R is a prime bi-ideal of R, P is a semiprime bi-ideal of R. Therefore $P_1 \subseteq P$ or $P_2 \subseteq P$. These implies that $\bar{\rho}(P_1) \subseteq \bar{\rho}(P)$ or $\overline{\rho}(P_2) \subseteq \bar{\rho}(P)$ which is a contradiction to our assumption. Hence $\bar{\rho}(P)$ is a prime bi-ideal of R.

(ii)similar to (i)

Corollary 3.5 Let ρ be a congruence relation on R and P be a prime bi-ideal of R. If $\underline{\rho}(P) \neq \emptyset$ then $\rho(A)$ is rough prime bi-ideal of R.

Proof. By Theorem 3.5 $\bar{\theta}(P).\underline{\theta}(P)$ are prime bi-ideal of R. Hence $\rho(P)$ is a rough prime bi-ideal of R.

Theorem 3.6. Let ρ be a congruence relation on R. If P is a semiprime bi-ideal of R then

(i) $\bar{\rho}(P)$ is a semiprime bi-ideal of R.

(ii) $\underline{\rho}(P)$ is a semiprime bi-ideal of R.

Proof: Straight forward.

Theorem 3.7. Let ρ be a congruence relation on R. If B is a irreducible bi-ideal of R then

(i) $\bar{\rho}(B)$ is a irreducible bi-ideal of R.

(ii) $\underline{\rho}(B)$ is a irreducible bi-ideal of R.

Proof: Let B be the irreducibe bi-ideal of R, then for any bi-ideals $B_1,\ B_2$ of R, $B_1 \cap B_2 = B$ implies either $B_1 = B$ or $B_2 = B$. -------------------------(1)

(i) Consider $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) = \bar{\rho}(B)$, From (1) , We have $\bar{\rho}(B_1) = \bar{\rho}(B)$ and $\bar{\rho}(B_2) = \bar{\rho}(B)$ Therefore $\underline{\rho}(B)$ be a irreducible bi-ideal of R.

Theorem 3.8. Every strongly irreducible semiprime bi-ideal of R is a strongly rough prime bi-ideals of R.

Proof. Let B be strongly irreducible semiprime bi-ideal of R. Since every strongly irreducible semiprime bi-ideal is irreducible semiprime bi-ideal of R. Then $\bar{\rho}(B)$ is irreducible semiprime bi-ideal of R. Let B_1 and B_2 be any two bi-ideals of R, then $\bar{\rho}(B_1)$ and $\bar{\rho}(B_2)$ are bi-ideals of R such that $\left(\bar{\rho}(B_1)\bar{\rho}(B_2)\right) \cap \left(\bar{\rho}(B_2)\bar{\rho}(B_1)\right) \subseteq \bar{\rho}(B)$. As

$\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right) \subseteq \bar{\rho}(B)$ and $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) \subseteq \rho(B_2)$, we have

$\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right)\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right) = \left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right)^2 \subseteq\ \bar{\rho}(B_1)\ \bar{\rho}(B_2)$

Thus $\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right)^2 \subseteq\ \bar{\rho}(B_1)\ \bar{\rho}(B_2)$

And $\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right)^2 \subseteq\ \bar{\rho}(B_2)\ \bar{\rho}(B_1)$.

This implies $\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right)^2 \subseteq\ \bar{\rho}(B_1)\bar{\rho}(B_2)\ \cap\ \bar{\rho}(B_2)\ \bar{\rho}(B_1) \subseteq \bar{\rho}(B)$.

Since $\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)$ is a bi-ideal and $\bar{\rho}(B)$ is a semiprime bi-ideal of R,

we have $\left(\bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\right) \subseteq \bar{\rho}(B)$.

Since $\bar{\rho}(B)$ is strongly irreducible, we have $\bar{\rho}(B_1) \subseteq \bar{\rho}(B)$ or $\bar{\rho}(B_2) \subseteq \bar{\rho}(B)$. This shows that $\bar{\rho}(B)$ is a strongly prime bi-ideal of R.

Similarly we prove $\underline{\rho}(B)$ is a strongly prime bi-ideal of R.

Hence $\rho(B)$ is a strongly rough prime bi-ideal of R.

Theorem3.9. Let ρ be a congruence relation on R and let B be a bi-ideal of R and $b \in R$ such that $b \notin B$. Then there exists a rough irreducible bi-ideal $\rho(I)$ of R such that $\rho(B) \subseteq \rho(I)$ and $b \notin \rho(I)$.

Proof.

Let B be a bi-ideal of R. Then by Theorem 3.3, $\bar{\rho}(B)$ is a a bi-ideal of R. Let X be the collection of all bi-ideal of R, which contains $\bar{\rho}(B)$ but does not contain b, since $\bar{\rho}(B) \epsilon X$, X is nonempty. The collection A is a partially ordered set under inclusion.

If Y is any totally ordered subset of X, then the union of all the subset in Y is a bi-ideal of R containing B and $b \notin Y$.

Hence by Zorn's Lemma there exists a maximal element $\bar{\rho}(I)$ in X. We show that $\bar{\rho}(I)$ is an irreducible bi-ideal of R.

Let $\bar{\rho}(L)$ and $\bar{\rho}(M)$ be two bi-ideals of R. Such that $\bar{\rho}(I) = \bar{\rho}(L) \cap \bar{\rho}(M)$. If both $\bar{\rho}(L)$ and $\bar{\rho}(M)$ properly contain $\bar{\rho}(I)$, then $b \in \bar{\rho}(L)$ and $b \in \bar{\rho}(M)$. Thus $b \in \bar{\rho}(L) \cap \bar{\rho}(M) = \bar{\rho}(I)$. This contradicts the fact that $b \notin \bar{\rho}(I)$ thus either $\bar{\rho}(I) = \bar{\rho}(L)$ or $\bar{\rho}(I) = \bar{\rho}(M)$.

Hence $\bar{\theta}(I)$ is irreducible bi-ideal of R.

Similarly we prove $\underline{\rho}(I)$ is a prime bi-ideal of R.

Thus $\rho(I)$ is a rough irreducible bi-ideal of R.

Theorem.3.10. For the semiring R, the following condition are equivalent.

i) R is both regular and intra-regular
ii) $\rho(B)^2 = \rho(B)$ for every bi-ideal B of R
iii) $\rho(B_1)\rho(B_2) \cap \rho(B_2)\rho(B_1) = \rho(B_1) \cap \rho(B_2)$ for any bi-ideals B_1, B_2 of R.
iv) Each bi-ideal of R is rough semiprime.
v) Each proper bi-ideal of R is the intersection of irreducible semiprime bi-ideal of R which contain it.

Proof.

(i)⇒(ii) Let R be both regular and intra-regular and B be a bi-ideal of R. $\bar{\rho}(B)$ is a bi-ideal of R. Then $(\bar{\rho}(B))^2 \subseteq \bar{\rho}(B)$ let $a \in \bar{\rho}(B)$. Since R is regular, there exists $x, y, z \in R = \bar{\rho}(R)$ such that $a\,x\,a$ and

$a = axa = ax(axa) = ax(\sum_{i=1}^{n} y_i\, aaz_i)xa$

$= \sum_{i=1}^{n} a(xy)_i\, aa(z_i x)a \in \bar{\rho}(B)\, R\, \bar{\rho}(B)\, \bar{\rho}(B)\, R\, \bar{\rho}(B) = \bar{\rho}(BRB)\, \bar{\rho}(BRB) \subseteq \bar{\rho}(B)\, \bar{\rho}(B) = \bar{\rho}(B)^2$

Thus $\bar{\rho}(B) \subseteq (\bar{\rho}(B))^2$.

Hence $\bar{\rho}(B) = (\bar{\rho}(B))^2$ for every bi-ideal B of R.

Similarly $\bar{\rho}(B) = (\bar{\rho}(B))^2$ for every bi-ideal of R.

Therefore $(\bar{\rho}(B))^2 = \rho(B)$.

(ii) ⇒(i) Let Q be a quasi-ideal of R, then Q is a bi-ideal of R . By Theorem 3.3, $\bar{\rho}(Q)$ is a bi-ideal of R. By hypothesis $(\bar{\rho}(Q))^2 = \bar{\rho}(Q)$. Thus R is both regular and intra regular semiring.

(ii) ⇒ (iii) Let B_1, B_2 be any two bi-ideals of R. By Theorem 3.3 , $\bar{\rho}(B_1)$ and $\bar{\rho}(B_2)$ are bi-ideal of R and $\bar{\rho}(B_1) \cap \bar{\rho}(B_2)$ is also a bi-ideal of R.

By hypothesis $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) = (\bar{\rho}(B_1) \cap \bar{\rho}(B_2))^2$

$= (\bar{\rho}(B_1) \cap \bar{\rho}(B_2)(\bar{\rho}(B_1) \cap \bar{\rho}(B_2)$

$\subseteq \bar{\rho}(B_1)\bar{\rho}(B_2)$

Similarly $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) \subseteq \bar{\rho}(B_2)\, \bar{\rho}(B_1)$.

Hence $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) \subseteq (\bar{\rho}(B_1)\, \bar{\rho}(B_2)\,) \cap (\bar{\rho}(B_2)\, \bar{\rho}(B_1))$.

Since $\bar{\rho}(B_1)\, \bar{\rho}(B_2)$ and $\bar{\rho}(B_2)\, \bar{\rho}(B_1)$ are bi-ideals of R. We have $(\bar{\rho}(B_1)\, \bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\, \bar{\rho}(B_1))$ is also a bi-ideal of R.

Then by hypothesis

$(\bar{\rho}(B_1)\bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\bar{\rho}(B_1)) =$

$(\bar{\rho}(B_1)\bar{\rho}(B_2)\,) \cap (\bar{\rho}(B_2)\, \bar{\rho}(B_1))\, (\bar{\rho}(B_1)\, \bar{\rho}(B_2)) \cap$

$(\bar{\rho}(B_2)\, \bar{\rho}(B_1))$

$\subseteq (\bar{\rho}(B_1)\, \bar{\rho}(B_2)\,)\, (\bar{\rho}(B_2)\, \bar{\rho}(B_1))$

$= \bar{\rho}(B_1)\, (\bar{\rho}(B_2)\,)^2 \bar{\rho}(B_1)$

$\subseteq \bar{\rho}(B_1)\, \bar{\rho}(B_2) \bar{\rho}(B_1)$

$\subseteq \bar{\rho}(B_1)\, R\, \bar{\rho}(B_1)$

$\subseteq \bar{\rho}(B_1)$

Similarly $(\bar{\rho}(B_1)\,\bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\,\bar{\rho}(B_1)) \subseteq \bar{\rho}(B_2)$
Hence $(\bar{\rho}(B_1)\,\bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\,\bar{\rho}(B_1)) = \bar{\rho}(B_1) \cap \bar{\rho}(B_2)$
It is also true for the bi-ideals of $\underline{\rho}(B_1)$ and $\underline{\rho}(B_2)$.
There fore $(\bar{\rho}(B_1)\,\bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\,\bar{\rho}(B_1)) = \bar{\rho}(B_1) \cap \bar{\rho}(B_2)$
(iii) ⇒(iv) Let B be the bi-ideal of R. We know that $\bar{\rho}(B)$ is a bi-ideal of R such that $(\bar{\rho}(B_1))^2 \subseteq \bar{\rho}(B)$ for any bi-ideal B_1 of R. Then by hypothesis, we have

$$\begin{aligned}\bar{\rho}(B_1) &= \bar{\rho}(B_1) \cap \bar{\rho}(B_1)\\ &= (\bar{\rho}(B_1)\,\bar{\rho}(B_1)) \cap (\bar{\rho}(B_1)\,\bar{\rho}(B_1))\\ &= (\bar{\rho}(B_1))^2 \subseteq \bar{\rho}(B).\end{aligned}$$

Which show that $\bar{\rho}(B)$is a semiprime bi-ideal of R.
Similarly we prove $\underline{\rho}(B)$ is a semiprime bi-ideal of R.
Therefore $\rho(B)$ is a rough semiprime ideal of R.
(iv) ⇒ (v) Let B be a proper bi-ideal of R. By Theorem 3.3 $\bar{\rho}(B)$ is a proper bi-ideal of R. Then $\bar{\rho}(B)$ is contained into the intersection of all irreducible bi-ideal of R which contains $\bar{\rho}(B)$. For the reverse inclusion let $a \,\epsilon\, \bar{\rho}(B)$. Then by Theorem [3.9] there exists an irreducible bi-ideal which contain $\bar{\rho}(B)$ does not contain a. This shows that $\bar{\rho}(B)$ is the intersection of all irreducible semiprime bi-ideal of R which contain it. Similarly $\underline{\rho}(B)$ is the intersection of all irreducible semiprime bi-ideals of R.

Hence each proper bi-ideal of R is the intersection of irreducible rough semiprime bi-ideal of R which contain it.
(v) ⇒ (ii) Let B be a bi-ideal of R. By Theorem $\bar{\rho}(B)$ is a bi-ideal of R. Then $(\bar{\rho}(B))^2$ is also a bi-ideal of R.
Thus by hypothesis

$$(\bar{\rho}(B))^2 = \cap_\alpha \{\bar{\rho}(B_\alpha) : B_\alpha \ is\ an\ irreducible\ semiprime\ bi-ideal\ of\ R \text{ such that } (\bar{\rho}(B))^2\ \bar{\rho}(B_\alpha) \text{ for all } \alpha\}$$

Since each B_α is semiprime, we have $\bar{\rho}(B) \subseteq \bar{\rho}(B_\alpha)$. Thus $\bar{\rho}(B) \subseteq \cap\ \bar{\rho}(B_\alpha) = (\bar{\rho}(B))^2$, but $(\bar{\rho}(B))^2 \subseteq \bar{\rho}(B)$ always holds.
Hence$(\bar{\rho}(B))^2 = \bar{\rho}(B)$ for each bi-ideal of R.
A similar proof is holds for bi-ideal $\underline{\rho}(B)$ of R. Hence $(\bar{\rho}(B))^2 = \bar{\rho}(B)$.

Theorem3.11. Let R be regular and intra-regular semiring then the following assertions are equivalents for a bi-ideal B of R
(i) ρ(B) is strongly rough irreducible
(ii) ρ(B) is strongly rough prime

Proof.
(i) ⇒ (ii) Let B be bi-ideal of R , then $\bar{\rho}(B)$ is a bi-ideal of R. By Theorem [ ] $\bar{\rho}(B)$ is semiprime, since $\bar{\theta}(B)$ is strongly irreducible, by Theorem [3.6], $\bar{\rho}(B)$ is strongly prime bi-ideal of R.
The proof of $\underline{\rho}(B)$ is strongly prime bi-ideal of R is similar. Thus $\rho(B)$ is strongly rough prime bi-ideal of R.
(ii) ⇒ (i) Let B be strongly prime bi-ideal of R and let B_1 and B_2 be any two bi-ideals of R. Then $\underline{\rho}(B_1)$ and $\underline{\rho}(B_2)$ are also bi-ideals of R such that $\bar{\rho}(B_1) \cap \bar{\rho}(B_2) \subseteq \bar{\rho}(B)$. Since R is regular and intra-regular by Theorem [ ]

$$(\bar{\rho}(B_1)\,\bar{\rho}(B_2)) \cap (\bar{\rho}(B_2)\,\bar{\rho}(B_1)) = \bar{\rho}(B_1)\ \cap\ \bar{\rho}(B_2)\ \subseteq \bar{\rho}(B).$$

Thus by hypothesis, we have $\bar{\rho}(B_1)\ \subseteq \bar{\rho}(B)$ or $\bar{\rho}(B_2)\ \subseteq\ \bar{\rho}(B)$.

Hence $\bar{\rho}(B)$ is strongly irreducible .

Similarly we prove $\underline{\rho}(B)$ is strongly irreducible.

Hence $\rho(B)$ is strongly rough irreducible.

CONCLUSION.

The theory of semirings and theory of rough sets have many application in various fields. Results of rough prime bi-ideals in Γ −semigroup can be extended to the general setting of semirings. We have bi-ideal introduced the notion of rough semiprime and rough irreducible bi-ideal of a semiring. The definition and results can be extended to other algebraic structures such as rings and modules.

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