UGC Approved Journal
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ISSN : 2348 - 5612 | Volume : 05 , Issue : 06 | April - June 2018

The Study of Properties of Linear Algebra and Matrices

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Abstract

Linear algebra is the branch of mathematics having a lot of application in various areas. Though a lot of advancement is done in mathematics still linear algebra is basic and important part of mathematics. In this paper, we will discuss about finding the vector, vector space and subspace of a vector, linear transformations and linear equations. Matrices also have vital role in solving linear equations. In this paper, we discussed about matrix, matrix multiplication and some important properties of matrices. Linear algebra and matrix have concrete application in quantum mechanics, control theory, analytic geometry and operator theory. It has extensive application in dimensionality reduction, changing 2-D vector spaces to 3-D vector spaces and much more.

Keywords: – Linear algebra, Linear spaces, Matrix, Matrix multiplication.

1 INTRODUCTION

The investigation of a vector is done in the beginning of linear algebra. A vector can be defined as an element of vector space having both a magnitude as well as a direction. It can be utilized in physical quantities like force, velocity and acceleration and they can be added to each other and duplicated with scalars, in a real vector space the line is illustration first. Thinking about the space of subjective and boundless measurement of current linear algebra has been reached out. A space having n component is called n-space in a vector space. A large portion of the valuable outcomes from 2-D and 3-D spaces can be stretched out into the higher dimensional spaces. Despite of the fact that individuals can only with significant effort envision vectors in n-space or n-tuples are valuable in addressing information. As n-tuple are lists of ordered n components, they can be easily summarized and data can be manipulated efficiently. For example, we can consider seven sister states of northeast India in tourism aspects, one can make and utilize, say, 7-D vectors or on the other hand 7-tuples to address terrorist activities of 7 states. One can choose to show terrorist activities of 7 states for a specific year, where the states structure is indicated, for model, (Meghalaya, Arunachal Pradesh, Mizoram, Assam, Manipur, Nagaland and Tripura), by utilizing a vector (r1, r2, r3, r4, r5, r6, and r7) where every state terrorist activity having in different situation. It is important for conceptual polynomial math which has a simply dynamic idea about that hypothesis are demonstrated in a vector space (linear space). A few distinct instances are gathering of invertible matrices or linear maps, and the circle of linear map of a vector space. Linear algebra additionally has a significant impact in examination, prominently, in the depiction of higher request subordinates in vector investigation, the investigation of variable quantities, and rotating maps. In this theoretical

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situation, the scalars are need not be number but a component of a vector space can be multiplied. As it is pre-requisite that the scalars structure a numerical design, known as field. The application of this field is normally field of complex or real numbers. Linear maps have components from one linear space to other (or itself), in a way that is viable with a scalar multiplication and expansion on the vector spaces. The arrangement of all this type of transformations is also a vector space. Sometimes the reason for vector space to be constant is that any linear transformation is addressed by a matrix. The itemized investigation of the calculations and the properties applied on matrices which include eigenvectors and determinants, is viewed as a component of linear algebra. We can say that straight issues of mathematics, those show linearity are those destined to settlement. For instance, differential math has an extraordinary arrangement for direct estimate of capacities; that the scalars structure a numerical design, known as field. The application of this field is normally field of complex or real numbers. Linear maps have components from one linear space to other (or itself), in a way that is viable with a scalar multiplication and expansion on the vector spaces. The arrangement of all this type of transformations is also a vector space. Sometimes the reason for vector space to be constant is that any linear transformation is addressed by a matrix. The itemized investigation of the calculations and the properties applied on matrices which include eigenvectors and determinants, is viewed as a component of linear algebra. We can say that straight issues of mathematics, those show linearity are those destined to settlement. For instance, differential math has an extraordinary arrangement for direct estimate of capacities. The difference from non-linear issues is gradually important. The overall strategy for tracking down a straightforward method for taking a gender on an issue, communicating it as linear algebra, and organizing it, if necessary, by matrix computations, is perhaps most appropriate in arithmetic.

1.1 Linear Algebra

The line from the beginning (dark blue) in R3 is a linear subspace, a typical topic in direct variable-based mathematics. Linear algebra-based math is a part of science worried about the investigation of vectors, vector spaces, linear transformation or linear maps and, matrices of linear equations. The vector spaces are focal subject in current science; subsequently, linear algebra is generally utilized in unique polynomial math as well as practical investigation. Linear algebra is also extensively illustrated in analytic geometry and is summarized in the administrative hypothesis. There are broad application areas in the physics and the economics, as non-linear models can be regularly approximated by linear ones.

2 BASICS OF LINEAR ALGEBRA

In beginning, linear algebra was applied only in the investigation of vectors in cartesian 2-D and 3-D. The quantity which have both magnitude and direction is called vector. Zero vector is an exemption vector with the zero size and without direction. Vectors can be utilized to address actual elements like force, velocity and acceleration. They can be added to one another and increased by scalars, subsequently framing the principal illustration of a vector space, where a qualification is made between" scalars", the real numbers for this position, and" vectors. Direct variable-based mathematics has reached the present day to think of discrete or infinite dimension spaces. A vector space of having n components is called a n-space. The vast majority of useful results from 2-D and 3-D can be reached from the higher dimensionality spaces. Despite the fact that one can only make significant effort in n-space with picture vectors, such vectors are valuable in addressing information. Since the vector, as n-tuples, is composed of n ordered tuples, the information in this structure can be efficiently summarized and handled. For example, to terrorist activities we can make and utilize, say, 7-tuples or 7-D(dimensional) vectors to address terrorist activities result of states. One can choose to show 7 states, where the states' structure is indicated, for instance, (Meghalaya, Arunachal Pradesh, Mizoram, Assam, Manipur, Tripura, and Nagaland), by utilizing a vector (r1, r2, r3, r4, r5, r6, and r7) where every state's is in its individual position.

3 IMPORTANT THEOREMS

- A basis is present in each vector space [1].
- The vector space with two basis as same number of elements that are proportional to size of vector space change.
- The necessary condition for any matrix to be invertible, is that the determinant of that matrix should be nonzero.
- Isomorphism linear map addressed by a matrix signifies the necessary condition for a matrix to be invertible.
- Existence of left inverse or right inverse of a square matrix shows that matrix is invertible.
- A matrix having eigen value greater than or equal to zero, is positive semidefinite.
- If all the eigen values of a matrix are greater than zero, than the matrix is positive definite.
- The necessary condition for a nxn matrix to be diagonizable is that it should have n linearly independent eigenvectors. For example, P is an invertible matrix and D is a diagonal matrix such that $A = (PDP)^{-1}$.
- According to the spectral theorem, if a matrix is symmetric in nature than the matrix is symmetrically diagonalizable.

4 LINEAR EQUATION

The equation having only first power of the variable and constant term is a linear equation. Linear equation has at least one factors. It happens bounteously in applied maths and also in some areas of mathematics. They emerge normally when displaying a lot of phenomena, these are especially valuable because more non-linear equations might be decreased in the form of linear equations by expecting that amounts of interest change to just a little extent from a few" foundation" state. When displaying a lot of phenomena that arise in general, they are especially valuable because more non-linear equations can be reduced to linear equations by hoping that the quantity of interest lies at some" foundation" position is changed to some extent. Linear equations have expected types. These types of article consider the instance of single equation

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ISSN : 2348 - 5612 | Volume : 05 , Issue : 06 | April - June 2018

for which one real solution. All of its substance applies for most part of linear equations and applied for complex solutions and solution in any field.

5 MATRICES

In mathematics, it is a matrix (more or less ordinarily matrices) which have representation of number in a rectangular form, which displayed at the right position. Matrix comprising just a single element or a row are known vectors, and for higher-dimensional, for example 3-D, varieties of numbers are known as tensors. Matrices are can be added, subtracted, and multiplied by a standard rule of linear transformations. These types of

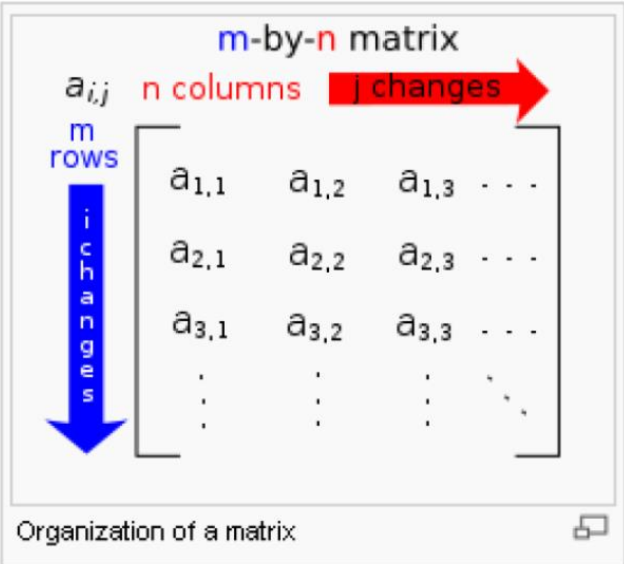

Fig.1. Organization of Matrix [12].

operation satisfy the identities expect multiplication isn't commutative that is AB ≠ BA. One utilization of matrices is to address higher-dimensional linear transform same as of linear functions of the structure f(y) = Ky, where K is a constant. Matrices can also monitor the coefficients in an arrangement of linear equations. For a square matrix, If the determinant and inverse exist, administer the conduct of answers for the following arrangements of linear equations. The eigen vessctor and eigen values give understanding of the related linear transformation. Matrices discover numerous utilizations. Material science utilizes them in different areas, for instance in mathematical optics and matrix mechanics. The last likewise prompted concentrating in matrices having infinite rows and columns. The distance encoding matrices of the bunch are centred in a diagram.

For example, a railway line project can be utilized in chart hypothesis, and PC designs use matrices for encoding projections of 3-D space to 2-D co-ordinates. Matrix calculus sums up old style scientific ideas like derivative or exponential of matrices. The last is widely used in addressing partial differential equations. Dodecaphonism and serialism are melodic developments of the 20th century which uses a square matrix to decide the example of music

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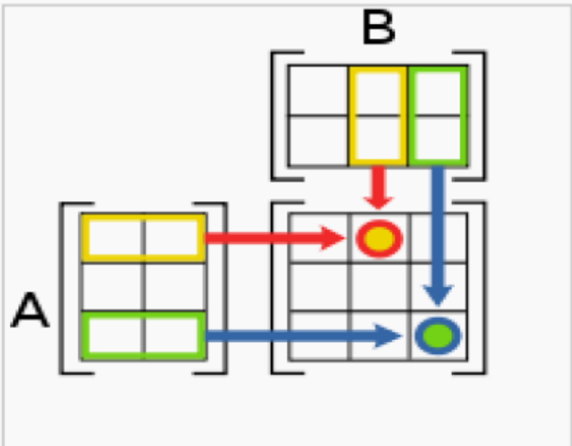
stretches. Because of their boundless use, impressive exertion has been made to foster effective techniques for matrix computation, especially if the matrices are huge. To this end, there are a few matrices disintegration techniques, which express matrices as results of different matrices with specific properties improving on calculations, both hypothetically and basically. The value of most of the elements in sparse matrices is zeros. It is applicable in mechanical examinations utilizing the finite component technique, regularly consider all the more explicitly custom-made calculations playing out these assignments. A former keyword makes close relationship



Schematic depiction of the matrix product **AB** of two matrices **A** and **B**.

Fig. 2 Product of two Matrix

of matrices with linear transformations. Different sorts of sections, like components in more broad numerical fields or even rings are likewise utilized.

$$[\mathbf{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \dots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j},$$

6 LINEAR EQUATION, LINEAR TRANSFORMATION AND MATRIX MULTIPLICATION

Multiplication of two matrices is conformable just if the number of columns of left matrix is equal to the number of rows of right matrix. In case A is a p*q matrix and B is a q*r matrix, then, at that point their matrix product AB is a p*r matrix whose sections are represented as: 1 _ I _ p and 1 _ j _ r. [5] For instance (the underlined passage 1 in the product is determined as the product 1 · 1 + 0 · 1 + 2 · 0 = 1). Matrix multiplication fulfils the guidelines

(XY)Z = X(YZ) (Associativity),
(X+Y) Z = XZ+YZ (Left distributivity)
Z(X+Y) = ZX+ZY (Right distributivity),

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at whatever point the dimensions of the matrices are with the end goal, that different products defined. [6] The product AB might be characterized without BA being characterized, specifically on the off chance that A and B are p*q and q*r matrices, separately, and p ≠ r. Regardless of whether the two products are characterized, they need not be equivalent, for

$$\begin{bmatrix} \underline{1} & \underline{0} & \underline{2} \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & \underline{1} \\ 2 & \underline{1} \\ 1 & \underline{0} \end{bmatrix} = \begin{bmatrix} 5 & \underline{1} \\ 4 & 2 \end{bmatrix}.$$

example, usually AB ≠ BA, i.e., matrix multiplication is not commutative, in check (real, or complex, rational), the difference of numbers whose product is independent of the size of the factors.

6.1 Linear Equations

A specific case of the matrix multiplication is firmly connected to linear equations. If x assigns a column vector (for example q×1- matrix of n factors x1, x2, ..., xn, and A is a p*q matrix, then, at that point the equation Ax = b, where b is a p×1 column vector, is identical with the arrangement of linear equations:

$$A_{1,1}x_1 + A_{1,2}x_2 + \ldots\ldots + A_{1,q}x_q = b_1$$
$$A_{p,1}x_1 + A_{p,2}x_2 + \ldots\ldots + A_{p,1}x_q = b_p.$$

According to matrices can be utilized to minimally compose and dealing with multiplication linear equations, for example arrangement of linear equations.

6.2 Linear Transformation

Matrices and multiplication of matrix uncover the fundamental highlights while identified with linear transformations, otherwise it is also called linear maps. Real p*q matrix A leads to a linear transform $R_q \rightarrow R_p$ mapping every vector x in R_q to product matrix A and vector x, which is a vector in R_p. Then again, every linear transform f: $R_q \rightarrow R_p$ emerges from a distinctive p*q matrix A: unequivocally, the (I, j) - section of A is i_{th} organize of f(ej), where ej = (0,...,0,1,0,...,0) is the unit vector with 1 in the j th position and 0 somewhere else. The matrix A is said to address the linear map f, and A is known as the transform matrix of f. The accompanying table shows various 2*2 matrices with the related linear maps of R2. The blue unique is mapped to the green line and shapes, the origin (0,0) is notice with a dark point.

7 CONCLUSIONS

In present day material science have key role in Linear transformations and the related field. Science utilizes matrices differently, especially for utilization of quantum physics for examination sub-atomic bonding and spectroscopy. Linear algebra and its application using matrix are introduced in this paper. Linear algebra consists of vector space, subspace, basis, matrices and their application. Vector space indicate the number of independent directions in vector space. A subspace as a part of matrix satisfying some particular set of condition such as set of matrices with rank two. The combination of different subspaces leads to the formation of a space. Every vector can be represented by n*1 or 1*n matrices. Linear algebra discusses about quantum mechanics and arrangement of different linear equation.

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