



DIMENSIONAL ANALYSIS : TO FIND ATOMIC BLAST RADIUS

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Abstract: Through this paper I would like to bring light on how Dimensional Analysis can be used to find the atomic blast radius.

ISSN : 2348-5612 © URR



Introduction: In engineering and science, **dimensional analysis** is the analysis of the relationships between different physical quantities by identifying their base quantities (such as length, mass, time, and electric charge) and units of measure (such as miles vs.kilometers, or pounds vs. kilograms vs. grams) and tracking these dimensions as calculations or comparisons are performed. Converting from one dimensional unit to another is often somewhat complex. Dimensional analysis, or more specifically the **factor-label method**, also known as the **unit-factor method**, is a widely used technique for such conversions using the rules of algebra.

The concept of **physical dimension** was introduced by Joseph Fourier in 1822. Physical quantities that are of the same kind (also called *commensurable*), have the same dimension (length, time, mass) and can be directly compared to each other, even if they are originally expressed in differing units of measure (such as inches and meters, or pounds and newtons). If physical quantities have different dimensions (such as length vs. mass), they cannot be expressed in terms of similar units and cannot be compared in quantity (also called *incommensurable*). For example, asking whether a kilogram is greater than, equal to, or less than an hour is meaningless.

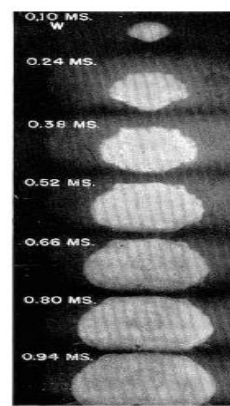
Any physically meaningful equation (and likewise any inequality and inequation) will have the same dimensions on its left and right sides, a property known as *dimensional homogeneity*. Checking for dimensional homogeneity is a common application of

dimensional analysis, serving as a plausibility check on derived equations and computations. It also serves as a guide and constraint in deriving equations that may describe a physical system in the absence of a more rigorous derivation

Dimensional analysis is a mathematical technique used to predict physical parameters that influence the flow in fluid mechanics, heat transfer in thermodynamics, and so forth. The analysis involves the fundamental units of dimensions MLT: mass, length, and time. It is helpful in experimental work because it provides a guide to factors that significantly affect the studied phenomena.

Dimensional analysis is commonly used to determine the relationships between several variables, i.e. to find the force as a function of other variables when an exact functional relationship is unknown. Based on understanding of the problem, we assume a certain functional form.

Historical Background: The first explosion of an atomic bomb was the Trinity test in New Mexico in 1945. Several years later a series of pictures of the explosion, along with a size scale, and time stamps were released and published in a popular magazine. Based on these photographs a British physicist named G. I. Taylor was able to estimate the power released by the explosion (which was still a secret at that time). How can the following pictures be



used to make this estimate?



DATA: Type of data available from photos: blast radius as a function of time

| | | | | |
|--------------------|---------|---------|---------|---------|
| Time t (sec) | 0.00038 | 0.00052 | 0.00066 | 0.00080 |
| Blast Radius R (m) | 25.4 | 28.8 | 31.9 | 34.2 |

One famous triumph of dimensional analysis was in finding a formula for the blast radius of an exploding atomic bomb as a function of time. Using this scaling was possible to make accurate estimates of the (highly classified) total energy of the US atomic weapon arsenal by looking at photographs of the blast release newspapers.

Let's find the relation for ourselves. Our first task is to identify the relevant physical quantities. For one, we are interested in the total energy of the bomb E , $[E] = ML^2 T^{-2}$ that must be one of our quantities. We're also interested in the radius of the blast r , and the time since the explosion t . So they too must be included in our scaling relation.

Thus far, we have

- explosive energy of the bomb E
- blast front radius r
- time since explosion t

We only need to add density of the medium into which bomb explodes, ρ to complete our set which has units of ML^{-3} . Thus we have

$$\rho^\alpha E^\beta r^\gamma t^\delta \sim 1$$

$$M^{\alpha+\beta} L^{-3\alpha+2\beta+\gamma} T^{-2\beta+\delta} \sim M^0 L^0 T^0 = 1$$



On solving the exponent relations, we find $\alpha=-\beta$, $\delta=2\beta$ and $5\beta=-\gamma$. We thus have three equations in four exponent variables, so we are free to choose the value of one of them.

Choosing $\alpha=1$, we find $\beta=-1$, $\delta=2$ and $\gamma=5$ that makes our scaling relation

$$\rho E^{-1} r^5 t^2 \sim 1$$

Thus the radius of the bomb front as a function of time must scale as

$$r \sim \sqrt[5]{\frac{Et^2}{\rho}}$$
$$r \sim t^{2/5}$$

CONCLUSION: Thus without knowing a single thing about physics of a bomb explosion under heavy atmosphere, we are able to obtain the correct scaling behavior for a bomb front as it explodes into a massive medium. Similarly there are other problems prevailing that can be solved by dimensional analysis.

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